CE 272
Traffic Network Equilibrium

Lecture 26

Sensitivity Analysis - Part II and Wrap-up

Previously on Traffic Network Equilibrium...

Bush-based methods essentially solve the following version of the TAP. The decision variables are x_{ij}^r, the link flows segregated by origins.

$$\min \sum_{(i,j)\in A} \int_0^{x_{ij}} t_{ij}(\omega)\, d\omega$$

$$\text{s.t.} \quad \sum_{j:(i,j)\in A} x_{ij}^r - \sum_{h:(h,i)\in A} x_{hi}^r = \begin{cases} \sum_{s\in Z} d_{is} & \text{if } i = r \\ -d_{ri} & \text{if } i = s \\ 0 & \text{otherwise} \end{cases} \quad \forall\, r \in Z$$

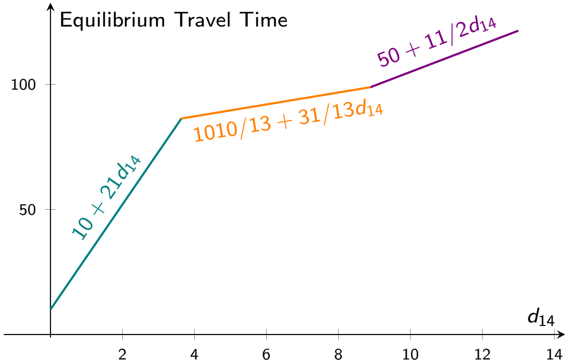
$$x_{ij} = \sum_{r\in Z} x_{ij}^r \,\forall\, (i,j) \in A$$

$$x_{ij}^r \geq 0 \,\forall\, (i,j) \in A, r \in Z$$

We will discuss two main components of the algorithm today and put these pieces together in the next class.

Previously on Traffic Network Equilibrium...

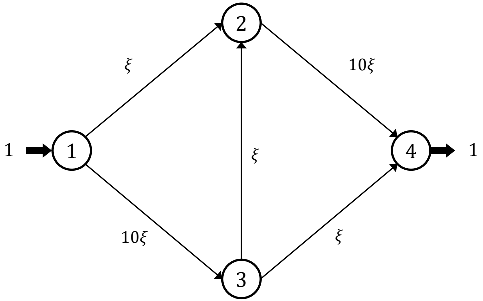
The plot has three regimes, each corresponding to different sets of used paths.



- The plot is a piecewise linear function. Is it always linear?
- The equilibrium travel times are differentiable within each of these pieces.
- At the degenerate points, the function is not differentiable and unused paths have same travel times as that of the used ones.

Previously on Traffic Network Equilibrium...

Thus, to compute the sensitivity to demand between $(1, 4)$, solve the UE problem in the following network.



Recall that all links are used in the original UE solution. If not, we should only use the equilibrium bush links to solve this new 'UE' problem.

Previously on Traffic Network Equilibrium...

The objective of the OD estimation problem is

$$\min f(\mathbf{d}, \mathbf{x}) = \eta \sum_{(r,s) \in Z^2} (d_{rs} - \hat{d}_{rs})^2 + (1 - \eta) \sum_{(i,j) \in \hat{A}} (x_{ij} - \hat{x}_{ij})^2$$

The derivative of this function with respect to d_{rs} is given by

$$\frac{\partial f}{\partial d_{rs}} = 2\eta(d_{rs} - \hat{d}_{rs}) + 2(1 - \eta) \sum_{(i,j) \in \hat{A}} (x_{ij} - x^*_{ij}) \frac{\partial x_{ij}}{\partial d_{rs}}$$

The values of $\frac{\partial x_{ij}}{\partial d_{rs}}$ can be obtained from the ξ values of the sensitivity analysis method.

Note that we need to solve a total of $|Z^2|$ equilibrium problems to obtain the gradient of the above function.

Lecture Outline

1. Sensitivity to Link Parameters
2. Network Design
3. Wrap-up

Lecture Outline

Sensitivity to Link Parameters

Sensitivity to Link Parameters

Introduction

Just as with demand, we can also model how the equilibrium travel times and flows change as a function of the link parameters.

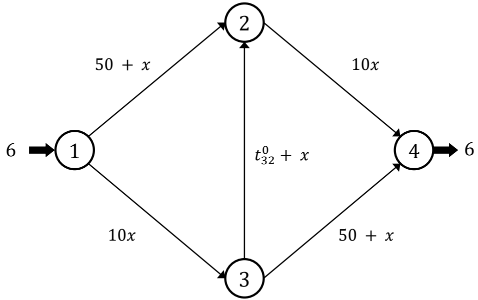
Typically, we are interested in changes to

- Free-flow speeds/Tolls and incentives
- Capacity

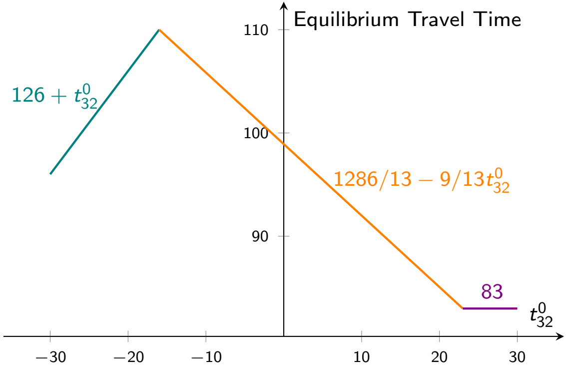
Sensitivity to Link Parameters

Introduction

Plot the variation in equilibrium link flows and travel times with t_{23}^0.



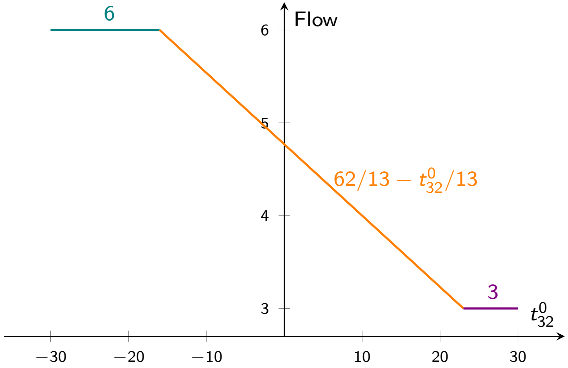
We let the value of t_{32}^0 to take negative values to model incentives.



- The equilibrium flows and travel times are again piecewise linear.
- We can see the Braess effect in play. As free-flow time decreases, i.e., as the middle link is improved, the equilibrium travel time increases!

Sensitivity to Link Parameters

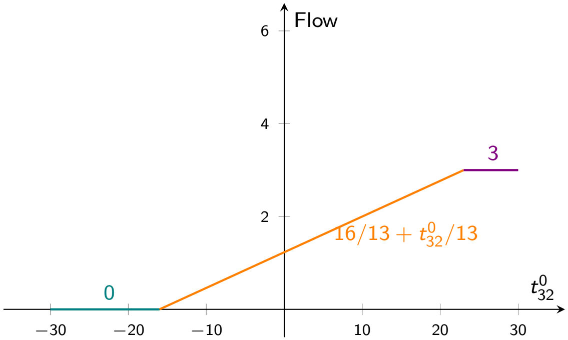
Introduction



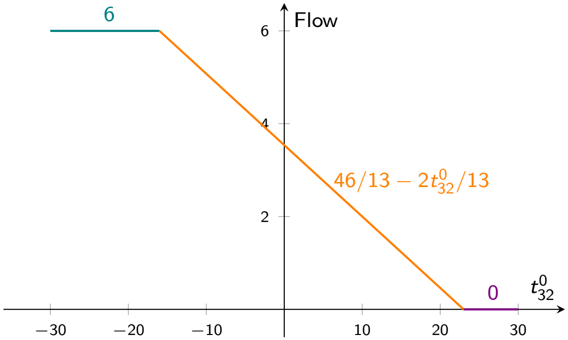
Flow on (1,3) and (2,4)

Sensitivity to Link Parameters

Introduction



Flow on (1,2) and (3,4)



Flow on (3,2)

Recall that the equilibrium solution to the bush based-formulation must satisfy

$$\mu_r^r = 0 \, \forall \, r \in Z$$
$$\mu_j^r - \mu_i^r - t_{ij}(x_{ij}) = 0 \, \forall \, (i,j) \in B^r, r \in Z$$
$$\sum_{j:(i,j)\in A} x_{ij}^r - \sum_{h:(h,i)\in A} x_{hi}^r = \begin{cases} \sum_{s \in Z} d_{is} & \text{if } i = r \\ -d_{ri} & \text{if } i = s \\ 0 & \text{otherwise} \end{cases} \quad \forall \, r \in Z$$
$$x_{ij}^r = 0 \, \forall \, (i,j) \notin B^r, r \in Z$$

When the link-parameters are perturbed, the bushes remain same. Hence, these conditions must hold for the new equilibrium values of x and μ.

Sensitivity to Link Parameters

Bush-based Approach

Suppose the free flow travel time on a link $(k, l) \in A$ changes. Let

$$\xi_{ij}^r = \frac{\partial x_{ij}^r}{\partial t_{kl}^0}, \Lambda_i^r = \frac{\partial \mu_i^r}{\partial t_{kl}^0}$$

Differentiate the following equations with respect to t_{kl}^0.

$$\mu_r^r = 0 \,\forall\, r \in Z$$

$$\mu_j^r - \mu_i^r - t_{ij}(x_{ij}) = 0 \,\forall\, (i, j) \in B^r, r \in Z$$

$$\sum_{j:(i,j)\in A} x_{ij}^r - \sum_{h:(h,i)\in A} x_{hi}^r = \begin{cases} \sum_{s\in Z} d_{is} & \text{if } i = r \\ -d_{ri} & \text{if } i = s \\ 0 & \text{otherwise} \end{cases} \quad \forall\, r \in Z$$

$$x_{ij}^r = 0 \,\forall\, (i, j) \notin B^r, r \in Z$$

Sensitivity to Link Parameters

Bush-based Approach

Suppose the free flow travel time on a link $(k, l) \in A$ changes. Let

$$\xi_{ij}^r = \frac{\partial x_{ij}^r}{\partial t_{kl}^0}, \Lambda_i^r = \frac{\partial \mu_i^r}{\partial t_{kl}^0}$$

The delay functions can be expressed as $t_{ij}(t_{kl}^0, x_{ij}(t_{kl}^0))$. The derivatives must therefore satisfy

$$\begin{aligned} \Lambda_r^r &= 0 \,\forall\, r \in Z \\ \Lambda_j^r - \Lambda_i^r - t'_{ij}(x_{ij}) \sum_{r' \in Z} \xi_{ij}^{r'} - \frac{dt_{ij}}{dt_{kl}^0} &= 0 \,\forall\, (i,j) \in B^r, r \in Z \\ \sum_{j:(i,j) \in A} \xi_{ij}^r - \sum_{h:(h,i) \in A} \xi_{hi}^r &= 0 \,\forall\, i \in N, r \in Z \\ \xi_{ij}^r &= 0 \,\forall\, (i,j) \notin B^r, r \in Z \end{aligned}$$

$\frac{dt_{ij}}{dt_{kl}^0}$ denotes the derivative of the delays wrt the free flow speed, while keeping x_{ij} constant. This is again a linear system of equations!

Sensitivity to Link Parameters

Bush-based Approach

Alternately, these form the KKT conditions of the following optimization model:

$$\min \sum_{(i,j)\in A} \int_0^{\xi_{ij}} \left(t'_{ij}(x_{ij})\xi + \frac{dt_{ij}}{dt^0_{kl}} \right) d\xi$$

$$\sum_{j:(i,j)\in A} \xi^r_{ij} - \sum_{h:(h,i)\in A} \xi^r_{hi} = 0 \, \forall \, i \in N, r \in Z$$

$$\xi^r_{ij} = 0 \, \forall \, (i,j) \notin B^r, r \in Z$$

As before, the optimal values of Λs can be obtained by solving a shortest path problem with fixed link travel times $t'_{ij}(x_{ij})\xi^*_{ij} + \frac{dt_{ij}}{dt^0_{kl}}$, where ξ^* is a solution to the above formulation.

Sensitivity to Link Parameters

Bush-based Approach

This problem can be treated as an 'equilibrium formulation' with linear link delay functions and 0 demand!

$$\min \sum_{(i,j)\in A} \int_0^{\xi_{ij}} \left(t'_{ij}(x_{ij})\xi + \frac{dt_{ij}}{dt^0_{kl}} \right) d\xi$$

$$\sum_{j:(i,j)\in A} \xi^r_{ij} - \sum_{h:(h,i)\in A} \xi^r_{hi} = 0 \, \forall \, i \in N, r \in Z$$

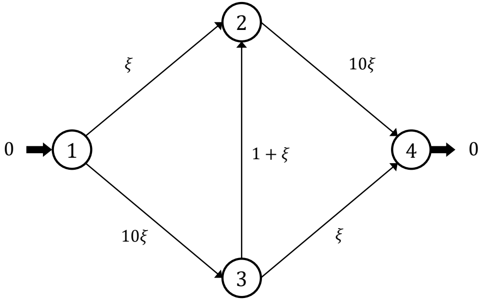
$$\xi^r_{ij} = 0 \, \forall \, (i,j) \notin B^r, r \in Z$$

As desired, there are no non-negativity constraints.

Sensitivity to Link Parameters

Bush-based Approach

Thus, to compute the sensitivity to free-flow speed on $(3, 2)$, solve the UE problem in the following network. Can we use Algorithm B?

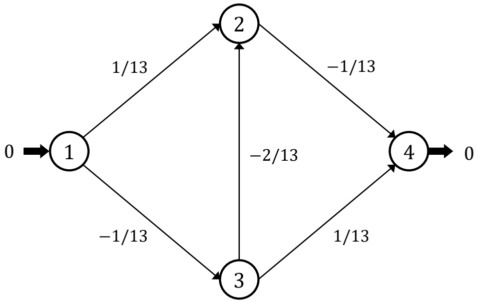


Recall that all links are used in the original UE solution. If not, we should only use the equilibrium bush links to solve this new ‘UE’ problem.

Sensitivity to Link Parameters

Bush-based Approach

The optimal ξ values can be obtained by equating the travel times on the three paths and are shown below. What is the 'equilibrium travel time' in the following network?



The 'UE travel time' in the above figure again corresponds to the slope of the piecewise linear function between UE travel time and free-flow time.

Lecture Outline

Network Design

Exactly similar results hold for sensitivity to capacity instead of free-flow travel times.

Knowing the derivatives with respect to link capacities can help us expand/design roadway segments.

First, re-define our delay functions as

$$t_{ij}(x_{ij}, z_{ij}) = t_{ij}^0 \left(1 + \alpha \left(\frac{x_{ij}}{C_{ij} + K_{ij} z_{ij}}\right)^{\beta}\right)$$

where z_{ij} is the amount (in ₹) spent on capacity expansion of link (i, j) and K_{ij} is the increase in capacity per ₹.

Network Design

Formulation

The objective is to then optimize the following problem.

$$\min f(\mathbf{z}, \mathbf{x}) = \gamma \sum_{(i,j) \in A} x_{ij} t_{ij}(x_{ij}, z_{ij}) + \sum_{(i,j) \in A} z_{ij}$$

$$\text{s.t. } \mathbf{x} \in \arg\min \sum_{(i,j) \in A} \int_0^{x_{ij}} t_{ij}(x, z_{ij}) dx$$

$$z_{ij} \geq 0 \, \forall \, (i,j) \in A$$

- γ is a parameter used to convert the system travel time into monetary units.
- We can also impose upper bounds on the total budget.
- For a given $\mathbf{z}$, the constraint provides an equilibrium link flow solution $\mathbf{x}$. Hence, we can write this as an optimization problem purely in terms of $\mathbf{z}$, i.e., $f(\mathbf{z}, \mathbf{x}(\mathbf{z}))$.

Just as with sensitivity to demand, the $\mathbf{z}$ values can be updated as

$$\mathbf{z} \leftarrow [\mathbf{z} - \eta \nabla f]^+$$

The derivative of the objective function $f(\mathbf{z}, \mathbf{x}(\mathbf{z})) = \gamma \sum_{(i,j)\in A} x_{ij} t_{ij}(x_{ij}, z_{ij}) + \sum_{(i,j)\in A} z_{ij}$ with respect to z_{ij} is given by

$$\begin{aligned} \frac{\partial f}{\partial z_{ij}} &= \gamma \left(\sum_{(k,l)\in A} \left(\frac{\partial f}{\partial x_{kl}} \frac{\partial x_{kl}}{\partial z_{ij}} \right) + x_{ij} \frac{dt_{ij}}{dz_{ij}} \right) + 1 \\ &= \gamma \left(\sum_{(k,l)\in A} \left(\frac{\partial x_{kl}}{\partial z_{ij}} \left(t_{kl}(x_{kl}, z_{kl}) + x_{kl} \frac{dt_{kl}}{dx_{kl}} \right) \right) + x_{ij} \frac{dt_{ij}}{dz_{ij}} \right) + 1 \end{aligned}$$

Solving the sensitivity analysis problem $|A|$ times, one for each link, gives us the partial derivatives involved in the above equation.

Lecture Outline

Wrap-up

Wrap-up

Frequently Asked Questions (FAQs) Revisited

What did you learn in this course this course?

- The primary objective of this course was to **predict route choice behavior** of travelers in large scale urban networks.
- There are several planning and operational problems that benefit from knowing travelers' route choices.

Don't we have software which can do that?

- Yes, software such as TransCAD and CUBE can model route choice behavior of travelers.
- Instead of using these software as black boxes, we tried to understand what's **behind the scenes**.
- You now have better insights into network modeling and can write your own code to solve real-world instances.
- You can add more features to existing models/formulate new ones that solve problems not addressed by current state-of-the-art tools.

What are you likely to take back from this course?

Hopefully, you have developed a habit for

- Identifying "themes" that are universal to multi-agent systems and a certain style of thinking in addressing related problems using mathematical models.
- Creating control mechanisms and optimizing resources for socially favorable outcomes.
- Finding the right combination of tractability and level of detail needed for a particular problem (there is *never* a one-size-fits-all solution).

Wrap-up

Frequently Asked Questions (FAQs) Revisited

What else are you likely to take back from this course?

- Writing KKT conditions and solving convex programs is a cakewalk. You've done this a zillion times now.
- Variational inequality is a traffic engineer's best friend.
- MSA is slow. Continental drift could beat it any day.
- If there is an intuitive answer to fix traffic, it is almost always wrong.
- MS Excel is not that bad after all.

What else are you likely to take back from this course?

Most importantly,

Wrap-up

Applications

Should a street be made one-way or two-way?

You are here: Home » City » B'luru is now City of one-ways

B'luru is now City of one-ways

Bengaluru, June 29, 2015, DHNS:

With 193 such roads, it stands second in the country Bosky Khanna

Bengaluru, referred to as the IT Capital and even Garbage City, has now earned a new tag - City of one-way roads.

Deccan Herald obtained data from RTOs and traffic management authorities of major Indian cities with a population of more than 75 lakh and vehicular population of over 30 lakh.

According to the data, the City has the second highest number of one-ways in the country. Mumbai stands first with 253 one-ways. According to the Mumbai traffic police and the Municipal Corporation, this is because of the ongoing Metro and Mono Rail construction and after the inclusion of Navi Mumbai. Many one-ways are also introduced during festivals.

BangaloreMirror

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TRAFFIC COPS MAY ADD TO CITY'S 193 ONE-WAYS

By Hemanth Kashyap, Bangalore Mirror Bureau | Oct 31, 2015, 04.00 AM IST

A- A+

Bangalore Traffic Police will review the functioning of one-ways in the city after suggestions were made that some were unwarranted, a few could be reversed and few roads could, in fact, be made into one-ways.

Police Commissioner N S Megharik has passed an order directing traffic police to take a new look at the 193 existing one-ways and make necessary corrections to decongest roads.

THE TIMES OF INDIA

CITY

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For Tech City, One-way the only way

TNN | Dec 17, 2004, 01.48 AM IST

A- A+

Bangalore has become a 'one-way' city — over 260 roads have been declared one-ways, resulting in high fuel bills and more pollution. But with over 25 lakh vehicles crowding city roads every day, only one-ways can keep the traffic moving, DCP Traffic (East) M.A. Saleem tells Azmath.

You have converted around 70 roads into one-ways in the last year. Critics say the city police have become fond of one-ways.

BENGALURU

Traffic woes: Are more one-ways the solution?

Imran Gowhar

BENGALURU APRIL 29, 2015 12:41 IST
UPDATED: APRIL 29, 2015 15:03 IST

SHARE ARTICLE | PRINT | A | A | A

The plan has already been drafted and also includes reversing the flow of traffic on some roads, such as Richmond Road and Residency Road.

Bumper-to-bumper traffic, the wait for signals to turn green and raised tempers caused by the time taken to cross important stretches in the heart of the city, are issues familiar to any Bengalurean. As a solution to these problems, the traffic police have proposed to implement the one-way rule on several roads in the central business district.

Where should you build new roads/expand capacity?
How would you design the roadway network of a new city?

Capacity Expansion

Amaravati Network Plan

What is the effect of tolls on congestion in urban areas?

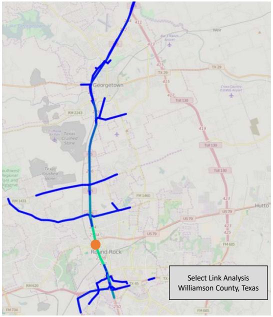
Bandra-Worli Sea Link

Singapore ERP System

London Cordon Pricing

Wrap-up
Applications

Who are ones using a given roadway link?



Select link analysis is a method that can help quantify **network-level impacts** of a local change. It has applications in congestion pricing and transportation equity.

The figure on the left shows set of links used by travelers taking the orange segment.

Wrap-up

Warning

Caution: In the future, you may use commercial software for the TAP. These are great tools and have neat GIS features but beware of their limitations.

Most packages may claim that they

- Solve equilibrium with multiple classes.
- Allow heterogeneity of travelers.

While these are true for the first three steps of the 4-step process, when they perform the equilibrium assignment, vehicles are converted to PCUs and VoT is averaged across the population.

Also, be aware that multiple path flow solutions exist. And never do network design using V/C ratios.

Wrap-up
Course Overview

The course was be grouped into three parts.

- **Part I: Introduction**
  In this part, we covered some background on convex optimization and studied shortest path algorithms and formulations for computing the equilibrium solution. We looked at link-based methods such as MSA and FW.
- **Part II: Variants**
  In this segment, we relaxed some of the assumptions to formulate and solve problems that are closer to reality. These included elastic demand, multiclass, bi-criterion assignment problems.
- **Part III: Advanced Topics**
  In this final part, we studied faster equilibrium algorithms such as gradient projection and Algorithm B and also sensitivity analysis and it's applications.

Wrap-up
What was Left Out

- Simplicial Decomposition and Other bush-based methods
- Complete analysis of bi-level optimization models and MPECs
- In-depth study of other TAP variants

However, you now have adequate background to read about any of these topics.

Dynamic Traffic Assignment: The models we saw in this course are called static traffic assignment models. They are mathematically attractive but they do not represent time dynamics.

- In reality, users depart at different times.
- Traffic builds and dissipates on a link in a more complex fashion and can create shockwaves. None of this is modeled in static TAPs.

Dynamic traffic assignment is an extension of the static approaches in which macroscopic traffic flow models are used instead of link delay functions.

Wrap-up

Advances in Network Modeling

The Wardrop principle in this setting states that ‘all users departing at the same time have equal and minimal travel times’.

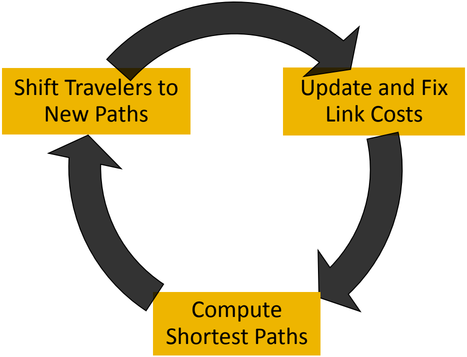
One can also formulate models in which users are allowed to switch their departure times.

However, existence and uniqueness of DTA is nearly impossible to establish because of non-linearity and discontinuities of the delay functions. DTA is built using a simulation framework and empirically most network converge.

Wrap-up

Advances in Network Modeling

The equilibrium procedure is similar but modifications are required at each step.



- **Update and Fix Link Costs:** This is carried out using simulators.
- **Compute Shortest Paths:** Since link delays vary across time, we now have to calculate time-dependent shortest paths.
- **Shift Travelers:** This step is typically done using gradient projection-like methods. Computing travel time derivatives is challenging.

Your Moment of Zen

36 Useful Proof Techniques: (Do not use it in my course!)

- **Proof by obviousness:** "The proof is so clear that it need not be mentioned."
- **Proof by general agreement:** "All in favor?..."
- **Proof by imagination:** "Well, we'll pretend it's true..."
- **Proof by convenience:** "It would be very nice if it were true, so..."
- **Proof by necessity:** "It had better be true, or the entire structure of mathematics would crumble to the ground."
- **Proof by plausibility:** "It sounds good, so it must be true."
- **Proof by intimidation:** "Don't be stupid; of course it's true!"
- **Proof by lack of sufficient time:** "Because of the time constrait, I'll leave the proof to you."
- **Proof by postponement:** "The proof for this is long and arduous, so it is given to you in the appendix."
- **Proof by accident:** "Hey, what have we here?!"
- **Proof by insignificance:** "Who really cares anyway?"
- **Proof by mumbo-jumbo:** $\forall \alpha \in \Phi, \exists \beta \ni \alpha * \beta = \varepsilon, \therefore$ ...
- **Proof by profanity:** (example omitted)
- **Proof by definition:** "We define it to be true."
- **Proof by tautology:** "It's true because it's true."
- **Proof by plagarism:** "As we see on page 289,..."
- **Proof by lost reference:** "I know I saw it somewhere...."
- **Proof by calculus:** "This proof requires calculus, so we'll skip it."

Source: http://jwilson.coe.uga.edu/emt668/emat6680.f99/challen/proof/proof.html

Your Moment of Zen

Contd.

- **Proof by terror:** When intimidation fails...
- **Proof by lack of interest:** "Does anyone really want to see this?"
- **Proof by illegibility:** 

- **Proof by logic:** "If it is on the problem sheet, it must be true!"
- **Proof by majority rule:** Only to be used if general agreement is impossible.
- **Proof by clever variable choice:** "Let A be the number such that this proof works..."
- **Proof by tessellation:** "This proof is the same as the last."
- **Proof by divine word:** "...And the Lord said, 'Let it be true,' and it was true."
- **Proof by stubbornness:** "I don't care what you say- it is true."
- **Proof by simplification:** "This proof reduced to the statement 1 + 1 = 2."
- **Proof by hasty generalization:** "Well, it works for 17, so it works for all reals."
- **Proof by deception:** "Now everyone turn their backs..."
- **Proof by supplication:** "Oh please, let it be true."
- **Proof by poor analogy:** "Well, it's just like..."
- **Proof by avoidance:** Limit of proof by postponement as it approaches infinity
- **Proof by design:** If it's not true in today's math, invent a new system in which it is.
- **Proof by authority:** "Well, Don Knuth says it's true, so it must be!"
- **Proof by intuition:** "I have this gut feeling."

Source: http://jwilson.coe.uga.edu/emt668/emat6680.f99/challen/proof/proof.html

The End