CE 269
Traffic Engineering

Lecture 20

Macroscopic Fundamental Diagrams

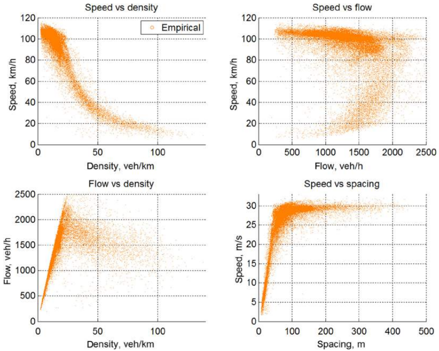
Previously on Traffic Engineering

Show that $o = lk$ in traffic where all vehicles have length l.

$$
\begin{aligned}
o &= \frac{1}{T} \sum_i (t_i^{off} - t_i^{on}) \\
&= \frac{1}{T} \sum_i l/v_i \\
&= l \frac{\Delta N}{T} \frac{\sum_i 1/v_i}{\Delta N} \\
&= l \frac{q}{v_s} = lk
\end{aligned}
$$

Previously on Traffic Engineering

The following is a picture from Ni (2016) with one year of traffic data from a city in US aggregated into 5-minute intervals.



The density values are calculated from the volume and speed measurements.

Lecture Outline

1. Macroscopic Fundamental Diagram
2. Extensions and Applications

Lecture Outline

Macroscopic Fundamental Diagram

Macroscopic Fundamental Diagram

Introduction

To analyze urban networks, we have both microscopic and macroscopic tools in our arsenal. The macroscopic ones, particularly, use simple triangular/trapezoidal fundamental diagrams.

Most of the literature on fundamental diagrams, however, focus on highway traffic. How would this data look for urban/city traffic?

This question led to the analysis of loop-detector data in Yokohama. Remember, with a single loop detector, it is easier to measure flow and occupancy.

Macroscopic Fundamental Diagram

Yokohama Experiment

Several previous studies tried to zoom out and analyze how average speed and flows were related in networks when aggregated as a single entity.

The results indicated that these were inversely related and the data mostly captured uncongested conditions.

Another set of aggregated variables which showed interesting trends were:

- Number of vehicles in the network (accumulation)
- Rate at which vehicles leave the network (trip completion rate)
- Production (average flow $\times$ trip lengths)

The first two can be viewed to be analogous to density and flux/flow.

Macroscopic Fundamental Diagram

Yokahama Experiment

Data collected in 2001 from 500 loop detectors aggregated at 5-min aggregation was used for this study. These typically measured q_i and o_i, the flow and occupancy, on link i.

Note $k_i = o_i/(\text{veh\_len})$, where (veh_len) is the average length of a car.

Let the set of arcs which contained a detector be A' and let A indicate the total number of arcs in the network.

In addition, 140 taxis fitted with GPS devices were also used to expand the inferences.

Suppose the length of link i is l_i. Define the weighted and unweighted averages of flow and occupancy/density as shown below:

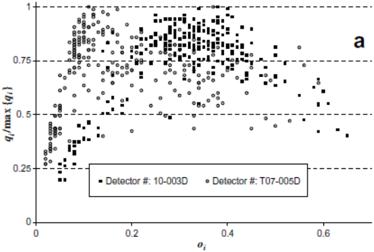
$$q^w = \frac{\sum_{i \in A'} q_i l_i}{\sum_{i \in A'} l_i} \qquad q^u = \frac{\sum_{i \in A'} q_i}{|A'|}$$

The production is defined as the numerator of $\sum_{i \in A'} q_i l_i$.

$$o^w = k^w(\text{veh\_len}) = \frac{\sum_{i \in A'} o_i l_i}{\sum_{i \in A'} l_i} \qquad o^u = k^u(\text{veh\_len}) = \frac{\sum_{i \in A'} o_i}{|A'|}$$

Macroscopic Fundamental Diagram
Yokohama Experiment

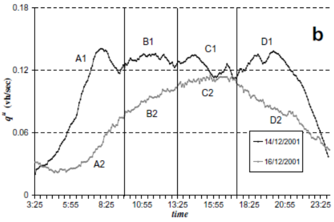
But the picture from individual detectors looked something like that shown in (a).

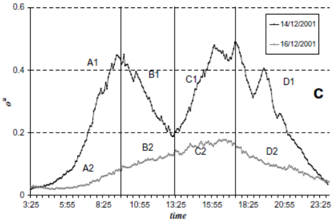


The scatter exhibits a lot of noise compared to those from highway loop detectors. (Why?)

What if the data from all detectors are aggregated? Would the scatter still persist.

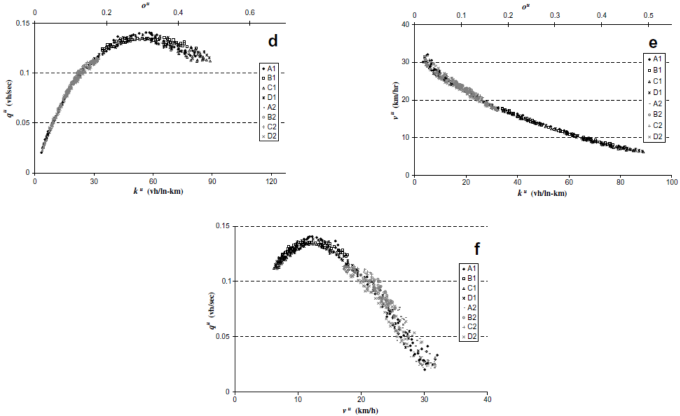
But first let's look at the unweighted average flows (left) and unweighted occupancy (right) across all detectors for a weekday and weekend.





The above plots indicate that the OD demand is likely different for these days in the network and further there is significant within-day variation A1–D2.

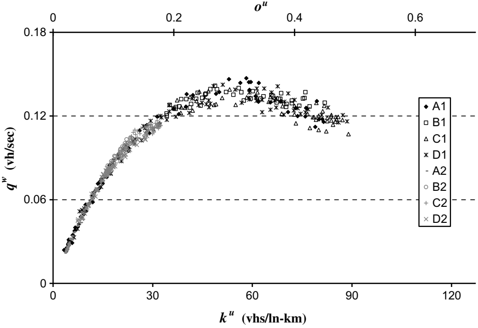
However, when aggregated, these unweighted quantities exhibit neat fundamental diagrams with very low scatter. Average speed v^u is simply set as q^u/k^u.



Macroscopic Fundamental Diagram

Yokahama Experiment

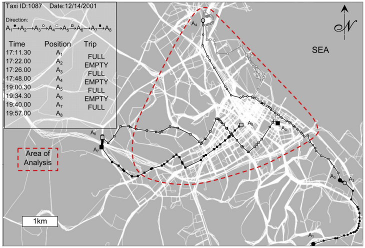
In fact, the weighted average flow vs. density/occupancy was also found to exhibit a fundamental diagram.



Importantly, the changes in OD patterns and time-of-day variations did not seem to affect these aggregate measures!

The empirical findings discussed so far apply to links in A'. Do they hold for the complete network?

To answer this, taxi data was used since we can observe trip completion rates, accumulation, and space-mean speeds easily.

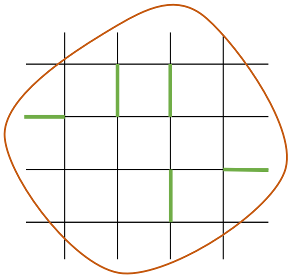


Some data cleaning was required to identify trip segments with passengers so that deadheading and search segments are avoided.

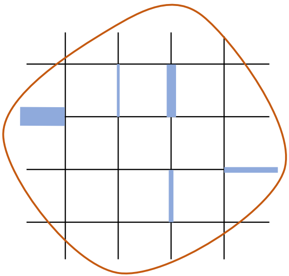
Macroscopic Fundamental Diagram

Taxi Data

To scale-up the data, loop detector flows in A' can be fused with the GPS data to get flows in A. (How?)

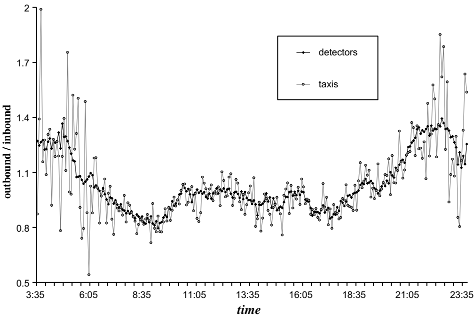


Sensor Locations



Taxi Flows on Links with Sensors

But is taxi data a good approximation of the overall demand?



The above plot of outbound/inbound traffic across the perimeter of A shows that taxi data is a good proxy for the rest of the traffic for most part of the day.

Macroscopic Fundamental Diagram

Taxi Data

For each $\Delta t = 5$ min interval, the following taxi data for the portion A was captured:

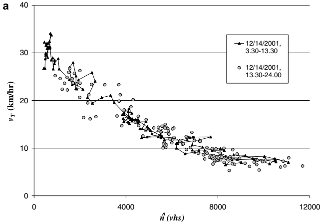
- Total distance traveled δ
- Total time spent τ ($\leq \Delta t$, why?)
- Space-mean speed $v_T = \delta/\tau$
- Number of taxis $n_T = \tau/\Delta t$
- Number of taxis that exited A, N_T
- Number of taxis that finished a trip in A, M_T

The last piece of information is unique since it gives an idea of the trip completion rate that cannot be calculated solely from point sensors.

How can we use this data to construct an MFD for the entire network?

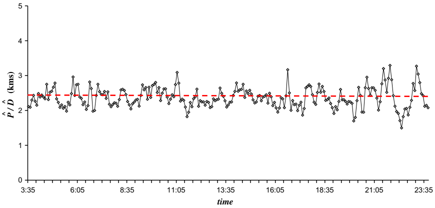
Measuring average density is difficult. Instead, the accumulation $n \approx kL$ can be used as a proxy, where L is the average length of trips.

This can be scaled up for all the vehicles by fusing the taxi and detector data.



The speeds of all cars can be assumed to be the same as the speeds of taxis.

Lastly, the productions and trip completion rates were found to have a nearly fixed ratio, equal to the average length of the trip.



The estimate of the productions of taxis, it equals $\delta/\Delta t$ since

$$\text{Productions} \approx qL = kv_T L = (n_T/L)(\delta/\tau)L = \delta/\Delta t.$$

The trip completion rates can be derived by adding N_T and M_T.

Extensions and Applications

All these findings were for one city. Can they be generalized and do they hold for other cities and demand patterns?

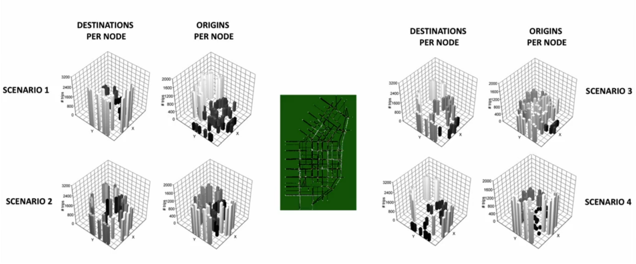
Initial results from simulations showed that these type of fundamental diagrams indeed exist for other networks.

https://youtu.be/_ANAbN15l1U

Using these simulations, MFDs were found in the following scenarios:

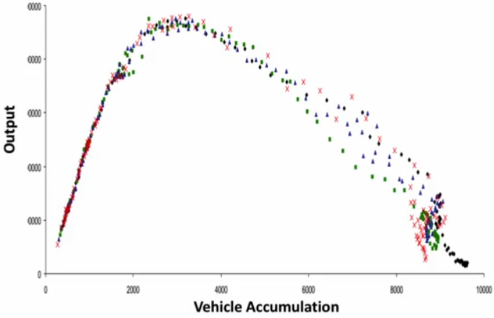
- The network was homogenously congested.
- Trip completion rate is proportional to the production.

The first scenario would still hold for a wide range of demand patterns.



Extensions and Applications

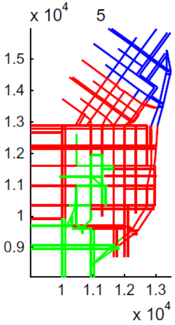
Existence

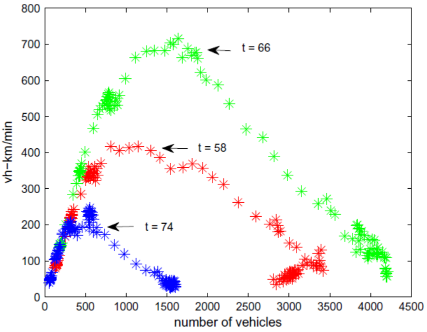


Extensions and Applications

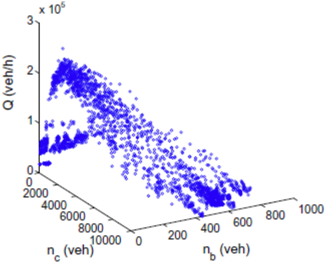
Clustering for Non-homogeneous Scenarios

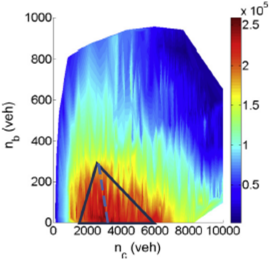
When the homogeneity assumption is violated, it may still be possible to divide the network into different regions based on some clustering methods and construct MFDs for each portion separately.





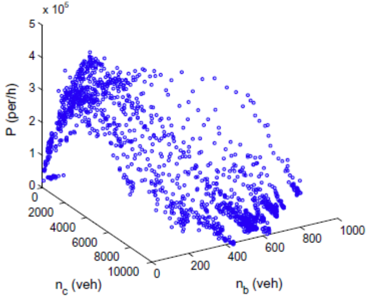
MFDs were also extended to multi-class settings involving two modes of travel: cars and buses.

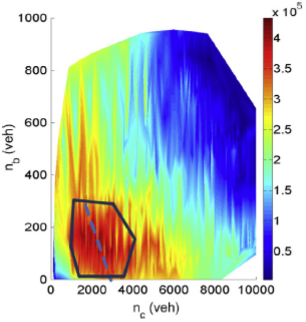




The trip completion rates are highest when there are no cars. However, this is metric is counting vehicles and not travelers.

Using average occupancy metrics for these modes, we can construct an equivalent MFD by measuring the passengers moved.





Extensions and Applications

Perimeter Control

The main application of MFDs is in perimeter control, where using a set of traffic lights on the boundary, we can make sure that a given area operates 'optimally'.

Specifically, the accumulation can be adjusted to maximize the trip completion rates using a simple single-reservoir model such as the ODE shown below.

$$\frac{dn}{dt} = q(t) - MFD(n(t))$$

These have been extended to the multi-reservoir case as well using coupled ODEs, where each portion could have its own MFD.

https://youtu.be/9ZML6zb4-R0

Additional Reading

Daganzo, C. F. (2007). Urban gridlock: Macroscopic modeling and mitigation approaches. Transportation Research Part B: Methodological, 41(1), 49-62.

Geroliminis, N., & Daganzo, C. F. (2008). Existence of urban-scale macroscopic fundamental diagrams: Some experimental findings. Transportation Research Part B: Methodological, 42(9), 759-770.

Daganzo, C. F., & Geroliminis, N. (2008). An analytical approximation for the macroscopic fundamental diagram of urban traffic. Transportation Research Part B: Methodological, 42(9), 771-781.

Your Moment of Zen