CE 269
Traffic Engineering

Lecture 18

Unsignalized Intersections

Previously on Traffic Engineering

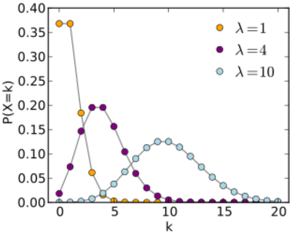
Definition

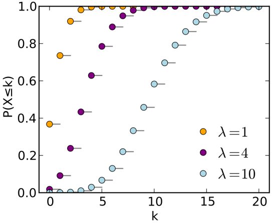
PMF of a Poisson distributed random variable with parameter $\lambda > 0$ is

$$\mathbb{P}(X = x) = p_X(x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

Claim

Suppose $X \sim Pois(\lambda)$, $\mathbb{E}(X) = V(X) = \lambda$





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Definition

Suppose $X \sim \exp(\lambda)$, its probability density function is defined as

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Claim

Suppose $X \sim \exp(\lambda)$*, its CDF is*

$$F_X(x) = 1 - e^{-\lambda x}$$

Check if $dF_X(x)/dx = f_X(x)$

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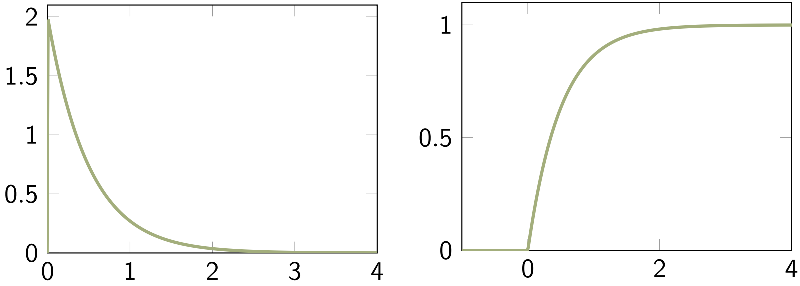
PDF and CDF of an exponentially distributed random variable with $\lambda = 2$ are shown below.



Claim

If $X \sim \exp(\lambda)$, *then* $\mathbb{E}(X) = 1/\lambda$ *and* $V(X) = 1/\lambda^2$

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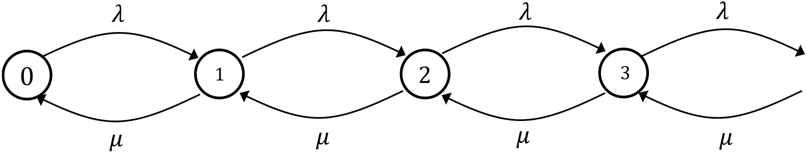
Let us now analyze a special case where the arrival process is a Poisson process with rate λ and the service times are exponential with rate μ. The fraction $\rho = \lambda/\mu$ is called the traffic intensity.



Can you write the global balance equations and find the steady state probabilities?

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Thus, the steady state probabilities are

$$p_0 = 1 - \rho$$

$$p_i = \rho^i (1 - \rho)$$

How do we find the expected length of the queue?

$$\mathbb{E}(L) = \sum_{j=0}^{\infty} j p_j = \sum_{j=0}^{\infty} j \rho^j (1 - \rho) = \frac{\rho}{1 - \rho}$$

We can also find the expected waiting time by defining new random variables and taking their expectations.

$$\mathbb{E}(T) = \frac{1}{\mu - \lambda}$$

How are $\mathbb{E}(L)$ and $\mathbb{E}(T)$ related? $\mathbb{E}(L) = \lambda \mathbb{E}(T)$. This is called Little's law and holds in more general settings.

Lecture Outline

1. Gap Acceptance
2. Interaction of Two Streams
3. Capacity and Level of Service

Lecture Outline

Gap Acceptance

Gap Acceptance

Introduction

Unlike signalized intersections which give define the right-of-way, drivers in navigating an unsignalized intersection must look for safe opportunities or gaps to enter the conflict area.

Additionally, traffic streams could have a hierarchy. Vehicles which have lower priority have to yeild to the ones with a higher priority (imagine a major arterial intersecting a minor road).

Typically, intersections on the minor street may have stop signs. This is called a two-way stop controlled (TWSC) junction. Alternately, if stops signs are present on all approaches, we call it an all-way stop controlled (AWSC) junction.

Of course, in this part of the world, all of these are just roads :)

https://youtu.be/UIthEM6pDqw

Gap Acceptance

Introduction

Usual questions of interest include:

- What gaps/headways (measured in time) are acceptable to drivers?
- What is the frequency with which these gaps occur?
- How many drivers will be able to depart within a given gap duration?
- What is the capacity of an unsignalized intersection?
- Can we quantify the performance of an unsignalized intersection using level of service measures?

Gap Acceptance

Critical Gap and Follow-up Time

The minimum gap that traffic in a minor stream accepts is called the *critical gap* and is denoted by t_c.

Note, we will use gap and headways interchangeably for this part of the course. So these are time elapsed between arrivals of the front ends or rear ends of consecutive vehicles at a point sensor.

When the gap in the major stream is longer than the critical gap, minor stream vehicles (if queued) enter at headways known as *follow-up time* t_f.

Gap Acceptance

Critical Gap and Follow-up Time

In reality, drivers need to be consistent or homogeneous. That is they can accept one gap at one junction and reject it at another.

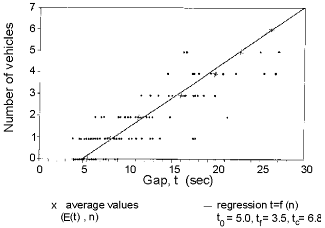
Also, different drivers have different thresholds for gap acceptance. Hence, distributions of these parameters are typically used.

Estimating t_c and t_f is critical to understanding the performance of the junction. These may depend on several factors such as the speed of major stream traffic and difficulty of maneuvers (going straight vs. turning).

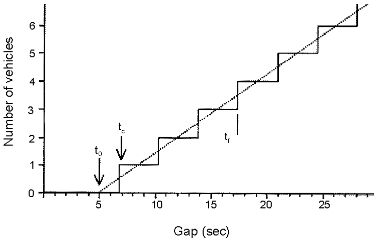
While measuring follow-up time is straightforward (why?), measuring critical gap is difficult since we can only measure the largest gap rejected and the accepted gap.

Two approaches are commonly used—regression and maximum likelihood.

Using real-world data, one can plot the gap size vs. number of drivers who accept that gap.



Data Used to Evaluate Critical Gaps and Move-Up Times (Brilon and Grossmann 1991).



The slope is t_f and if the x-intercept is t_0, then the critical gap is (Why?)

$$t_c = t_0 + t_f/2$$

Gap Acceptance

Maximum Likelihood Approach

In the maximum likelihood approach, the critical gap is assumed to be log-normal and we assume that for each user the following data is available:

- a_i logarithm of accepted gap of driver i (∞ if no gap was accepted)
- r_i logarithm of the largest gap rejected by driver i (0 if no gap was rejected)

The goal is to find estimates of the parameters of the log-normal distribution μ and σ^2.

Is the log-normal distribution a good choice? Isn't it common to assume that the gaps are exponentially distributed?

Gap Acceptance

Maximum Likelihood Approach

Suppose f and F represent the pdf and cumulative distribution function of the associated normal distribution, write the log-likelihood function and maximize it.

$$\begin{aligned}\mathcal{LL} &= \ln\left(\prod_{i=1}^{n} F(a_i) - F(r_i)\right) \\ &= \sum_{i=1}^{n} \ln\left(F(a_i) - F(r_i)\right)\end{aligned}$$

Using $\frac{\partial F(x)}{\partial \mu} = -f(x)$ and $\frac{\partial F(x)}{\partial \sigma^2} = -\frac{x-\mu}{2\sigma^2} f(x)$, set the gradient of $\mathcal{LL}$ to zero and solve for μ and σ.

The expected critical gap is

$$\mathbb{E}(t_c) = \exp(\mu + 0.5\sigma^2)$$

Gap Acceptance

Headway Distributions

Headways on the major street are usually assumed exponentially distributed. Suppose there were 456 vehicles passed through a junction in 1 h, what is the expected number of headways greater than 5 s?

If instead there were 1440 vehicles, what is expected number of headways less than 0.1 s?

The exponential distribution does a good job for low traffic but can overestimate short headways. Hence, two other distributions are commonly used—shifted exponential and bunched exponential.

Gap Acceptance

Headway Distributions

The shifted or displaced exponential guarantees a minimum headway t_m by defining the complementary CDF in the following way:

$$\mathbb{P}(h > x) = \begin{cases} \exp(-\mu(x - t_m)), & \text{if } x \geq t_m \\ 1, & \text{otherwise} \end{cases}$$

What is the mean of this random variable?

Often, traffic appears in platoons. To model such streams, the bunched exponential distribution could be used

$$\mathbb{P}(h > x) = \begin{cases} \alpha \exp(-\mu(x - t_m)), & \text{if } x \geq t_m \\ 1, & \text{otherwise} \end{cases}$$

For this distribution, the minimum headway t_m is maintained with probability $1-\alpha$ and with α, the distribution takes the form of a shifted exponential distribution.

This results in platoons that are t_m apart with exponential gaps between the bunches. What is the probability that there are n vehicles in a platoon?

These are geometrically distribution with pmf

$$\mathbb{P}(N = n) = (1 - \alpha)^{n-1}\alpha$$

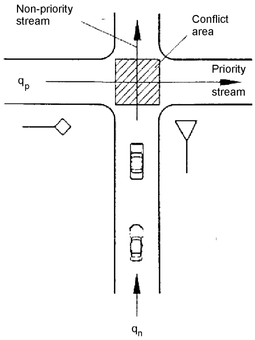
What is the average number of vehicles in the platoon?

Interaction of Two Streams

Consider a simple intersection with two one-way streams

Suppose the average volume in the priority and non-priority streams be q_p and q_n, respectively.

Let $f(t)$ be the pdf of gaps in the major stream and let $g(t)$ be the number of vehicles from the minor stream that can enter the major stream with a gap of t.



The capacity of the major stream traffic can be written as

$$q_m = q_p \int_0^{\infty} g(t) f(t) dt$$

The function $f(t)$ can either take the form of exponential, shifted or bunched exponential. For estimating $g(t)$, we could use the knowledge of t_c and t_f like the regression plots described earlier.

$$g(t) = \begin{cases} 0 & \text{if } t < t_0 \\ \frac{t-t_0}{t_f} & \text{otherwise} \end{cases}$$

where $t_0 = t_c - \frac{t_f}{2}$

We could also use the expected value of g treating the number of vehicles entering a gap of duration t as a random variable.

$$g(t) = \sum_{n=1}^{\infty} n p_n(t)$$

where $p_n(t)$ is the probability that n minor stream vehicles enter a gap of duration t.

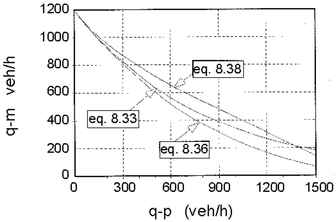
For the step-function discussed in the regression model, p can be written as

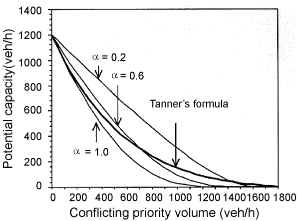
$$p_n(t) = \begin{cases} 1 & \text{if } t_c + (n-1)t_f \leq t < t_c + nt_f \\ 0 & \text{otherwise} \end{cases}$$

These two approaches yield the following formulate for capacity of the minor stream assuming the major approach headways are exponential.

$$q_m = q_p \frac{\exp(-q_p t_c)}{1 - \exp(-q_p t_f)}$$

$$q_m = \frac{1}{t_f} \exp(-q_p t_c)$$





Other equations in the above figure relax the assumptions of constant t_f and t_c and exponential headways.

Interaction of Two Streams

Queuing Model

Another line of approach is to use a queuing model to estimate capacity and delays.

For example the Pollaczek-Khinchine formula can be used for the M/G/1 queue with Poisson arrivals and general service distribution (gap acceptance process).

$$L = \rho + \frac{\rho^2 + \lambda^2 Var(S)}{2(1-\rho)}$$

$Var(S)$ is the variance of the service time S. Little's law can be used to get the waiting time or the delay $W = L/\lambda$. Capacity is the inverse of the service time.

Lecture Outline

Capacity and Level of Service

Capacity and Level of Service

Indo-HCM

The Indo-HCM provides a procedure for estimating the capacity of minor approaches and LoS for unsignalized intersections with 3 or 4 approaches.

The approaches used for data collection either have 2 or 4 lanes.

Unlike the earlier example, an unsignalized intersection can have several minor movements which squeeze through gaps of major movements.

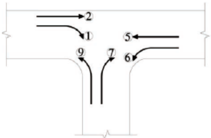


Figure 8.8 *(a)*: Vehicular Movements at a Typical Three-Legged intersection

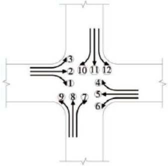


Figure 8.8 *(b)*: Vehicular Movements at a Typical Four-Legged intersection

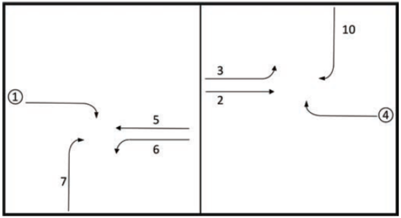
Table 8.2: Priority Ranks for Different Movements

Priority Rank	Movement
1	Movement 2
	Movement 3
	Movement 5
	Movement 6
2	Movement 1
	Movement 4
3	Movement 7
	Movement 10
4	Movement 8
	Movement 11

Capacity and Level of Service

Calculating Conflicting Volume

As usual, we convert flows into PCUs. The conflicting volume calculations depend on the priority of the movements.



For two-lane major streets,

$$V_{c,1} = 1.5v_5 + v_6 + v_7$$

$$V_{c,4} = 1.5v_2 + v_3 + v_{10}$$

Capacity and Level of Service

Calculating Conflicting Volume

Rank	Movement	Conflicting Flow *(per hour)*	
		Two lane major street	Four lane major street
1	v_2 v_3 v_5 v_6	--	--
2	v_1 v_4	$1.5v_5 + v_6 + v_7$ $1.5v_2 + v_3 + v_{10}$	v_5 v_2
3	v_7 v_{10}	$v_4 + v_5 + v_1 + v_2$ $v_1 + v_2 + v_4 + v_5$	$v_4 + v_5 + v_1 + 0.5v_2$ $v_1 + v_2 + v_4 + 0.5v_5$
4	v_8 v_{11}	$v_4 + v_5 + v_1 + v_2 + v_3 + v_{10}$ $v_1 + v_2 + v_4 + v_5 + v_6 + v_7$	$v_4 + v_5 + v_1 + v_2 + v_{10}$ $v_1 + v_2 + v_4 + v_5 + v_7$

Capacity and Level of Service

Base Critical Gap

Movement	Vehicle Type: Two-Wheeler	Auto rickshaw	Standard Car	Big Car	LCV	Bus	TAT / MAT
Four lane divided intersection							
Right turning from major to minor street	2.5	2.7	2.7	2.9	3.3	3.6	3.8
Right turning from minor to major street	3.5	3.7	3.8	4.1	4.9	5.5	5.7
Through traffic on minor	5.8	5.9	6.8	7.6	7.9	7.9	8.6
Two lane undivided intersection							
Right turning from major to minor street	2.9	3.2	3.5	3.9	-	-	-
Right turning from minor to major street	3.2	3.5	3.8	-	-	-	-
Through traffic on minor	3.5	4.2	4.9	-	-	-	-

Capacity and Level of Service

Adjusted Critical Gap

These critical gaps are adjusted based on the vehicle composition (particularly, taking large vehicles into account).

$$t_{c,x} = t_{c,base} + f_{LV} \ln(P_{LV})$$

Movement	Vehicle Type						
	Two-Wheeler	Auto rickshaw	Standard Car	Big Car	LCV	Bus	TAT / MAT
Four lane divided intersection							
Right turning from major to minor street	0.25	0.27	0.46	0.55	0.48	0.53	0.74
Right turning from minor to major street	0.61	0.64	0.88	0.93	0.86	0.84	0.85
Through traffic on minor	0.34	0.38	0.58	0.45	0.44	0.44	0.27
Two-lane undivided intersection							
Right turning from major to minor street	0.38	0.63	0.78	1.02	-	-	-
Right turning from minor to major street	0.07	0.07	0.01	-	-	-	-
Through Traffic on minor	0.07	0.07	0.07	-	-	-	-

$$C_x = a \times V_{c,x} \frac{e^{-V_{c,x}(t_{c,x}-b)/3600}}{1-e^{-V_{c,x}t_{f,x}/3600}}$$

Where,

C_x = capacity of movement 'x' *(in PCU/h)*,
$V_{c,x}$ = conflicting flow rate corresponding to movement x *(PCU/h)*,
$t_{c,x}$ = critical gap of standard passenger cars for movement 'x' *(s)*,
$t_{f,x}$ = follow-up time for movement 'x' *(s)*, and
'a' and 'b' = adjustment factors based on intersection geometry.

Major Street Configuration	Adjustment Factors	Subject Movement		
		Right Turn from Major	Right Turn from Minor	Through on Minor
Four-lane divided	a	0.80	1.00	0.90
	b	1.30	2.16	5.04
Two-lane undivided	a	0.70	0.80	1.10
	b	-0.11	0.72	0.72

Capacity and Level of Service

Level of Service

Level of Service	Volume-Capacity ratio
A	< 0.15
B	0.16 - 0.35
C	0.36 - 0.55
D	0.56 - 0.80
E	0.81 - 1.00
F	> 1.00

Additional Reading

Troutbeck, R.J., & Brilon, W. (1997). Unsignalized Intersection Theory. FHWA Report.

Your Moment of Zen