Equilibrium Analysis of Low-Conflict Network Designs

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ABSTRACT

Low-conflict network designs aim to reduce intersection delay by restricting or eliminating crossing conflicts. These designs range from alternating one-way street grids in central business districts, to more radical designs which eliminate crossing conflicts altogether. However, travel distances in such networks are generally higher than in traditional networks. This paper proposes an equilibrium approach for evaluating the tradeoff between increased distance and reduced intersection delay in networks of varying topology and demand patterns. To accomplish this, suitable link performance functions are developed to reflect different types of intersection control. We compare three control strategies: two-way grids, one-way grids, and a vortex design with priority merges. These strategies are compared in grid networks, with sensitivity analysis to demand levels and other parameters. The vortex-based design generally leads to lower average travel times and higher trip distances. However, at very high demand levels the use of gap acceptance formulas for priority merges, together with route choice, results in unstable, chaotic conditions.

1 INTRODUCTION

A recent paper by Eichler et al. *(1)* introduces the idea of designing entire urban networks without crossing conflicts, proposing that such designs may improve both safety and mobility in the right setting. These authors cataloged a number of intersection and network designs adhering to this philosophy, some resembling familiar geometries, while others are of a more radical nature (for instance, involving driving on the opposite side of the road as usual for certain blocks). The price of removing such conflicts is typically an increase in the shortest network distance between an origin and a destination, since eliminating crossing conflicts requires more circuitous routing. These authors performed an analysis of typical distance increases, finding that an additional 2–4 blocks of travel are required, depending on the modeling assumptions used.

The seemingly-paradoxical idea of increasing mobility by eliminating turning options at intersections suggests a tradeoff between increased travel times due to longer routes, and decreased travel times due to reduced intersection delay. This paper aims to study these tradeoffs at the macroscopic level, considering both the dependence of intersection delay on route flows, and the route choice process as drivers choose least-time paths. To accomplish this goal, suitable delay models must be developed for intersections using traditional controls, as well as the novel designs with only merging conflicts.

The primary contribution of this paper is the introduction of a network equilibrium framework which can be used to evaluate the performance of these novel design strategies, as compared to traditional ones. Thus, this paper builds on the analysis of Eichler et al. *(1)*, who only considered distance increases without congestion. Developing this framework requires specifying delay functions representing signalized and unsignalized control schemes. As we show, gap-acceptance merging models for unsignalized intersections lead to a nonseparable equilibrium problem which may have multiple solutions. Furthermore, we demonstrate that the interaction between route choice and gap-acceptance can lead to surprising results: while average vehicle travel distances and times are remarkably stable over a wide range of demand values, demand in excess of a high threshold value can trigger network-wide deterioration in travel times. In some ways, this phenomenon resembles "gridlock," a phenomenon that is not typically seen in static assignment. This phenomenon highlights the importance of incorporating network equilibrium with gap-acceptance merging models to evaluate these networks, and could not have been observed without fully including both in the modeling framework.

The remainder of the paper is organized as follows: Section 2 reviews the concept of low-conflict designs. Section 3 then describes the equilibrium formulation we propose to evaluate such designs, and caution that merge-based designs may admit multiple equilibria. Section 4 then compares low-conflict and traditional designs in networks of varying topology and demand levels, and Section 5 discusses in particular the stability of vortex-based designs under high demand. Finally, Section 6 concludes and discusses future directions.

2 LOW-CONFLICT INTERSECTIONS

It is well-known that intersection delay is the predominant cause of congestion on arterial streets. Traditional intersection designs, based on stop or signal control, are relatively inefficient and only allow a fraction of the total arriving demand to move simultaneously. This occurs because of conflict points at which one stream must yield to another, either based on a signal indication, gap acceptance, or based on the "rules of the road" (for instance, in most countries that drive on the right, if two vehicles arrive at an unsigned intersection, priority is given to the vehicle approaching from the right). Therefore, considerable attention has been given to eliminating conflict points. Crossing conflicts in particular are more dangerous, and require signalized control at lower volumes. The lost time associated with signalized control increases delays still further.

There are several well-known intersection and network designs which attempt to reduce or eliminate conflict points. Many urban grid networks are built upon one-way streets that alternate direction, greatly reducing the number of conflict points at intersections, and allowing each intersection to be controlled by a two-phase signal regardless of traffic volumes. At a single intersection, the roundabout design eliminates crossing conflicts entirely, replacing one central intersection with small merge/diverge intersections at each approach. More recent intersection designs, such as diverging diamonds or "super streets," further attempt to reduce or eliminate crossing conflicts.

Eichler et al. *(1)* extend this philosophy to an entire network. Their paper presents a number of intersection designs and network patterns which ensure network connectivity without requiring any intersection with a crossing conflict. Figure 1 shows three specific network designs they propose. The left design consists of concentric vortex rings which alternate between clockwise and counterclockwise flow, while the middle design consists of vortices with a wider spacing. In these designs, flow is always permitted around the block in either the clockwise or counterclockwise direction; essentially, each block serves as a large roundabout, and the intersections provide connection points between these roundabouts. The rightmost design is a "hybrid" design which incorporates a few signalized intersections in a primarily conflict-free network.

The evaluation method described in this paper is based on the network equilibrium concepts first established by Pigou *(2)*, Wadrop *(3)*, and Beckmann et al. *(4)*. Specifically, a static traffic assignment problem is formulated with delay arising from either signals or yielding behavior at merging conflicts in the vortex design. In the latter case, this results in an asymmetric equilibrium, which necessitates a variational inequality formulation *(5, 6, 7)*. Unfortunately, the use of gap acceptance concepts for delay functions results in a nonmonotone cost mapping, as is demonstrated in the following section. Most algorithms for asymmetric equilibrium problems assume some flavor of monotonicity, such as quasi, strict, pseudo, or strong *(8, 9, 10, 11, 12)*. Therefore, the algorithms suggested in these papers cannot be proven to converge for this problem, and multiple equilibria may result. Instead, we use the much simpler method of successive averages, which despite its simplicity and known slow convergence, nevertheless produces acceptable results for the small networks and aggregate measures of effectiveness used in this paper.

3 MODEL

This section describes the network and equilibrium model we use to evaluate low-conflict intersection designs *vis-á-vis* traditional ones. This model takes the form of a static network assignment problem. In transportation networks, nodes typically represent intersections and arcs represent roadway links. When turning movements must be distinguished, one approach is to "explode" the intersection node by approach, as in the middle panel of Figure 2. Another alternative, shown in the right panel, is to introduce a "dual" representation, in which the nodes in the dual graph represent the *arcs* in the original graph (that is, the physical roadway links), and arcs represent permissible turning movements[1]. Mathematically, if the original, "primal" graph $\hat{G}(\hat{N}, \hat{A})$ has node and arc

[1] While "dual graph" has more than one meaning in the graph theory literature, we use this term in the sense of Eichler et al. *(1)*

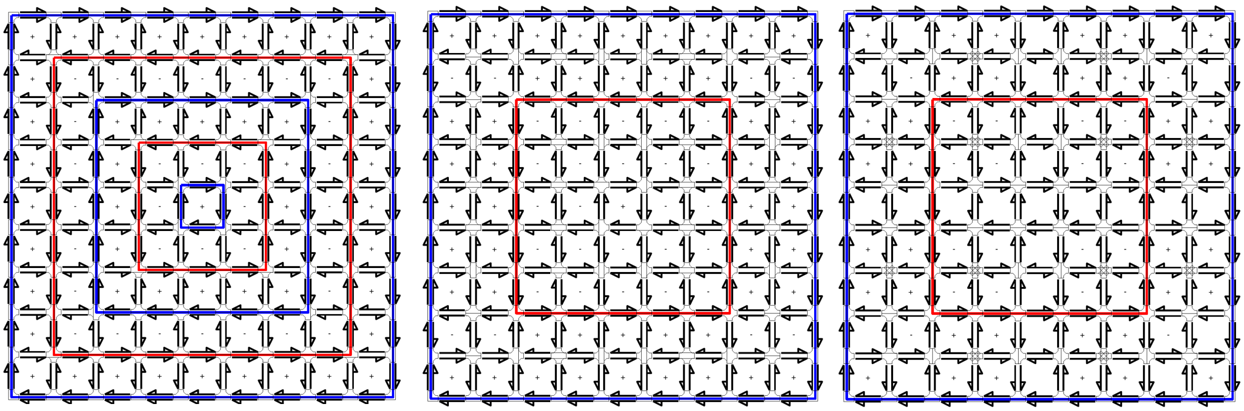

FIGURE 1 Three crossing conflict-free network designs from Eichler et al. *(1)*

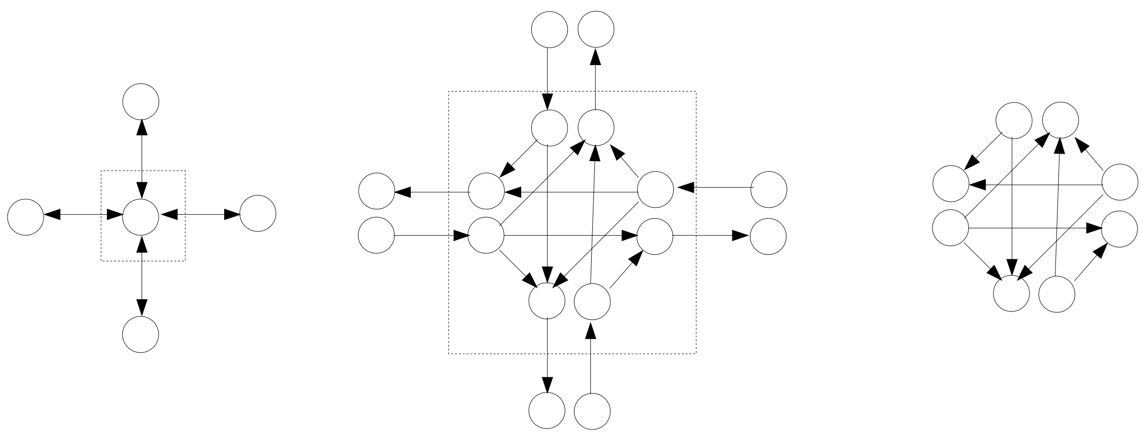

FIGURE 2 Standard intersection representation (left panel), exploded (middle), and dual representation (right)

sets $\hat{N}$ and $\hat{A}$, respectively, the dual graph $G = (N, A)$ is defined by $N = \hat{A}$ and $(a, b) \in A$ iff drivers are permitted to turn from arc a onto arc b. In this paper, the dual graph is the more convenient representation, and will be adopted exclusively from here — from this point, nodes will refer to physical roadway links, and arcs will refer to turning movements between links.

This paper considers three network topologies: two-way grids, one-way grids, and vortices. In the grids of two-way and one-way streets, each intersection is controlled by a traffic signal, and all possible turning movements are allowed. However, due to a reduction in crossing conflicts, the one-way grid involves only two signal phases, while the two-way grid involves four-phase signals. The vortex topology can be seen in the left panel of Figure 1, and consists of concentric one-way thoroughfares which alternate between the clockwise and counterclockwise directions. Each block allows unimpeded travel in either the clockwise or counterclockwise direction, depending on the orientations of the bordering thoroughfares. The (dual) representations of these graphs are shown in Figure 3. This figure only shows networks of three blocks by three blocks to save space; the evaluation in Section 4 uses larger networks following the same patterns.

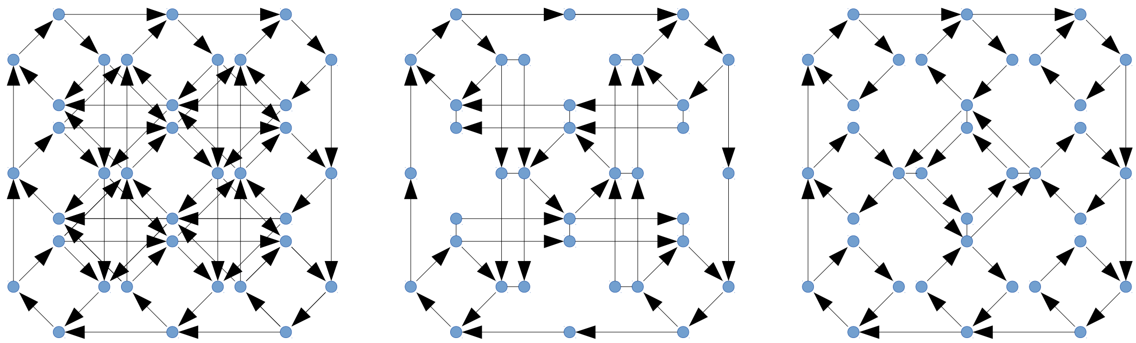

FIGURE 3 Two-way grid (left panel), one-way grid (middle), and vortex (right) networks

Because the focus of this paper is evaluating basic design patterns, rather than optimizing to specific conditions, these networks contain extensive symmetry. Every link will have identical saturation flow, and demand will be uniformly distributed between every possible pair of nodes in the network. In particular, note how multiple-lane facilities are described in each network. Each arc represents an independent lane of travel. Where lane changing is permitted, zero-cost artificial arcs are created joining two nodes, as shown in Figure 3.

3.1 Delay calculations

To focus on the effects of intersection control, we assume that delays occur due to queuing at intersections. Therefore, the travel time t_{ab} on each link (a, b) is divided into two parts: a constant free-flow time t_a^0 associated with traversing the physical roadway segment associated with node a, and a flow-dependent delay $D_{ab}(x)$ associated with the turning movement (a, b). That is,

$$t_{ab}(x) = t_a^0 + D_{ab}(x) \tag{1}$$

This formulation implicitly assumes that turning movements do not obstruct each other (for instance, if there are separate turn lanes of sufficient length). Below we show how D_{ab} is calculated for low-conflict and traditional signalized intersections.

3.1.1 Signalized intersections

Delay at signals follows traditional principles of traffic engineering, as in the Highway Capacity Manual *(13)*. Assuming protected phasing, let G_{ab} and C_{ab} denote the effective green time and cycle length for the turning movement (a, b) and the corresponding intersection. Further let s_{ab} reflect the saturation flow of this turning movement, and $X_{ab} \equiv x_{ab}C_{ab}/s_{ab}G_{ab}$ the degree of saturation. If the analysis period is of length Λ, the average delay for this turn movement is

$$D_{ab}(x_{ab}) = \frac{C_{ab}}{2}\left(\frac{(1 - G_{ab}/C_{ab})^2}{1 - \min\{X_{ab}, 1\}G_{ab}/C_{ab}}\right) + \frac{\Lambda}{4}\left(X_{ab} - 1 + \sqrt{(X_{ab} - 1)^2 + 8X_{ab}C_{ab}/s_{ab}G_{ab}\Lambda}\right) \tag{2}$$

where the first term represents the uniform delay, and the second term the incremental delay due to stochastic arrivals. Note that this delay function is increasing and only depends on the flow on movement (a, b).

In this paper, two-way grids are controlled using a four-phase signal, with protected left turns alternating with through and right movements from the north-south and east-west approaches. A uniform cycle length C is adopted at each intersection. This cycle length is divided equally between the north-south and east-west approaches; furthermore, the proportion of each approach's time allotted to protected left turns is identical. Sensitivity to these parameters is investigated in the numerical results. Intersections in one-way grids, by contrast, are controlled using two-phase signals which allot the cycle length equally between the two approaches.

3.1.2 Low-conflict intersections

Merges at low-conflict intersections are assumed to have yield control. In this case, the approach with priority is designated the *primary approach*, while the approach which must yield is the *secondary approach*. These priorities may be determined by signage, channelization, or driving regulation and custom. Consider a merge with primary approach (a, c) and secondary approach (b, c). Since approach (b, c) must yield to (a, b), the flows from a are not delayed and $D_{ac} \equiv 0$. However, vehicles on approach b must yield to those on approach a, and therefore D_{bc} depends on the flows on both primary and secondary approaches x_{ac} and x_{bc}.

Since there are no traffic signals to interrupt flow, the assumption of Poisson arrivals (and thus exponentially-distributed headways) on both approaches seems reasonable. In traffic operations, it is customary to define a *critical gap* t_c and a *follow-up gap* t_f at unsignalized streams. The critical gap is the minimum headway in the primary stream for which a vehicle in the secondary stream will turn. Given that the gap is large enough for one vehicle turn, the additional headway needed for each additional vehicle is specified by the follow-up gap. Experimentally, t_f is somewhat smaller than t_c.

Under these assumptions, the capacity of the secondary approach is

$$C_{bc} = \frac{x_{ac} \exp(-x_{ac} t_c)}{1 - \exp(-x_{ac} t_f)} \tag{3}$$

and the average delay to vehicles on the secondary approach is

$$D_{bc}(x_{ac}, x_{bc}) = \frac{1}{C_{bc}} + \frac{\Lambda}{4} \left[\frac{x_{bc}}{C_{bc}} - 1 + \sqrt{\left(\frac{x_{bc}}{C_{bc}} - 1 \right)^2 + \frac{8 x_{bc}}{C_{bc}^2 \Lambda}} \right] \tag{4}$$

where Λ is the length of the analysis period.

As shown in Section 3.2, the delay mapping implied by this function does not have a positive-definite Jacobian, and multiple equilibria may arise. In theory, this complicates the process of comparing alternative designs. However, as shown in Section 4, this effect does not seem to play a significant role in the designs considered, except possibly at very high demand levels.

3.2 Equilibrium formulation

Let d^{rs} be the demand for travel from node r to node s. Also let π be a path in G, h^π be the flow on path π, and Π^{rs} be the set of paths connecting node r to node s. The travel time on path π

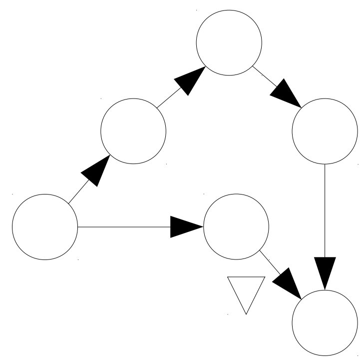

FIGURE 4 Network to demonstrate equilibrium nonuniqueness.

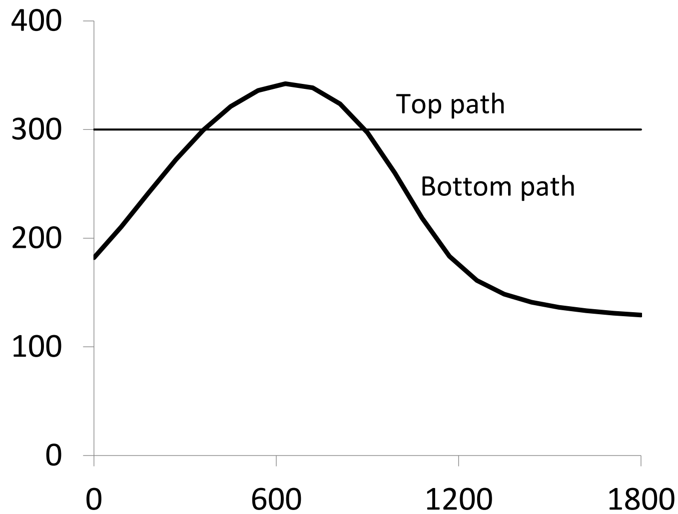

FIGURE 5 Path travel times as flow on priority approach varies.

is denoted T_π, and is the sum of the travel times on each link in π. We consider the equilibrium solution in which every used path connecting every origin and destination has equal and minimal cost. The set H represents each feasible traffic assignment, that is

$$H = \left\{ \mathbf{h} : \sum_{\pi \in \Pi^{rs}} h^\pi = d^{rs} \forall (r,s) \in N^2, \mathbf{h} \geq \mathbf{0} \right\} \tag{5}$$

A feasible path flow vector $h^* \in H$ is an equilibrium iff it solves the variational inequality

$$\mathbf{T}(\mathbf{h}^*) \cdot (\mathbf{h} - \mathbf{h}^*) \geq 0 \qquad \forall \mathbf{h} \in H \tag{6}$$

The presence of asymmetric link interactions implies that no equivalent convex programming formulation exists *(14)*. Further, the mapping $\mathbf{T}(\mathbf{h})$ is nonmonotone, and hence multiple equilibria can occur. Consider the example in Figure 4, which has two paths. The bottom path is shorter, but is the secondary approach at the merge with the longer top path. Specifically, let each link have unit free-flow time. The only delayed link is the one marked with the yield sign (∇) in the figure; this link's delay is given by (4). Assume a one-hour analysis period, and a total assigned volume of 1800 vehicles, with a critical gap of 4 seconds and a follow-up gap of 2 seconds.

Figure 5 shows how the travel times on the two paths vary according to the number of vehicles choosing the top (longer) path. Evidently, there are three equilibria: (1) zero vehicles on the top path (in which case all vehicles have a travel time of 182 seconds); (2) 362 vehicles on the top path (all vehicles have travel time of 300 seconds); and (3) 892 vehicles on the top path (all vehicles have travel time of 300 seconds). The first and third of these are stable with respect to small perturbations in route choice, while the second is unstable, and any perturbation is likely to lead to further route switching in that direction. Figure 5 also clearly establishes the nonmonotonicity of $\mathbf{T}$ using the gap-acceptance formulas; for instance, let $\mathbf{h_1} = \begin{bmatrix}0 & 1800\end{bmatrix}$ and $\mathbf{h_2} = \begin{bmatrix}600 & 1200\end{bmatrix}$ (indexing the top path first and the bottom path second). Then $\mathbf{T}(\mathbf{h_1}) = \begin{bmatrix}300 & 180\end{bmatrix}$ and $\mathbf{T}(\mathbf{h_2}) = \begin{bmatrix}300 & 341\end{bmatrix}$, and $(\mathbf{h_1} - \mathbf{h_2}) \cdot (\mathbf{T}(\mathbf{h_1}) - \mathbf{T}(\mathbf{h_2})) \approx -96600 < 0$.

Most exact algorithms for solving asymmetric equilibrium problems and general variational inequalities rely on some form of monotonicity for the cost mapping $\mathbf{T}$, and therefore may not converge. For this reason, we use the method of successive averages *(15)*. While the convergence of this algorithm is infamously slow, the networks used in this paper are of sufficiently small size that it provides adequate convergence. Furthermore, as shown in Section 4, the measures of interest (average travel distance and time) are quite stable and converge quickly, at least in the symmetric networks under consideration.

4 NETWORK EVALUATION

The networks used for evaluation represent a square region of 81 blocks (10 north-south streets and 10 east-west streets), with the topologies as shown in Figure 3. Each block is 100 ft long, and each link has a free-flow travel time of 10 seconds. The following parameters were used as to establish a "base case" around which sensitivity analysis would be performed: the analysis period is one hour long; total demand among all OD pairs is 8100 vph (100 vehicles per block, or a vehicle leaving each block every 36 seconds), uniformly distributed. For both one-way and two-way grids, the signal cycle length is 60 seconds. Each link has a saturation flow of 1900 vph *(13)*. For two-way grids, the protected dual-left turn phase is 5 seconds long. No lost time is assumed, an assumption which leads to a slight underestimation of delay in the signalized networks. For the vortex network, a critical gap and follow-up gap of 4 seconds and 2 seconds are assumed.

The method of successive averages algorithm was implemented in the C programming language. This algorithm was initialized in three distinct ways in an attempt to detect multiple equilibria, if they exist. These methods include initializing all origins with an all-or-nothing assignment to the free-flow shortest path tree; loading origins sequentially, updating travel times after each loading occurs; and loading origins sequentially in reverse order by ID. All three initialization schemes always converged to the same equilibrium solutions, which are reported in the next section. While this is only a cursory examination of this issue, these results do suggest some stability in the equilibria reported below.

The two measures of effectiveness are the *average trip distance* (ATD) and *average trip time* (ATT). ATD is obtained by multiplying the flow on each link by the length of the link, summing across links, and dividing by the total travel demand; ATT is obtained by multiplying the flow on each link by its travel time (including both free-flow and delay), and then summing and dividing as

TABLE 1 Average distances and times for the base networks.

Network	ATD (ft)	ATT (min)
Two-way signal	1144	3.70
One-way signal	1208	3.13
Vortex	1435	1.72

with ATD. Table 1 shows the results for the base network. As might be expected, travel distances are shortest in the two-way grid (which provides more direct connections), somewhat longer in the one-way grid, and longer still in the vortex network; while the opposite trend is seen with respect to travel times. In this way, the tradeoffs between direct connections and reduced intersection delay can be quantified and made explicit.

These rankings are relatively stable as demand varies; Figure 6 plots ATD and ATT as the demand varies from 900 total trips to 16200 total trips (respectively corresponding to a vehicle leaving each block every 5.4 minutes, and every 18 seconds). In particular, the travel distances associated with the one-way signal grid are nearly constant, indicating that congestion levels are not high enough to warrant route switching even as delays increase. For the two-way grid, trip distances increase slightly as demand increases, because the equilibrium condition is spreading demand over some paths of longer length than the free-flow shortest paths. Over this range of demand values, the vortex design exhibits a high level of stability in both ATD and ATT, with a barely perceptible increase in ATT. ATT increased from approximately 102 seconds to 107 seconds over the range of demand values tested; typical delays when merging onto a vortex link are on the order of 2–3 seconds, because traffic flows are low enough that most gaps in the vortex stream are acceptable.

Although the focus of this paper is on network evaluation, and not optimization, the sensitivity of the results to the specific signal timings chosen should be studied. Figure 7 shows how ATD and ATT vary as the cycle length changes, using horizontal lines to indicate the values for the base case. While the ATD values change for the two-way signal grid, the ranking of the three designs is stable across the range of cycle lengths considered. Likewise, although the specific ATT values vary with the cycle length, the ranking of the three designs does not change. Similar results were obtained by varying the length of the left turn phase, although these plots are not included for reasons of space.

The use of the method of successive averages suggests that the convergence rate of the measures of effectiveness should be specified, since this algorithm is known to be quite slow. Convergence of equilibrium-based algorithms are typically based on a gap measure, such as the relative gap. This paper uses the average excess cost (AEC) as a gap measure; as defined in Boyce et al. *(16)*, this is the average difference between a traveler's experienced travel time, and the shortest path travel time available to him or her. Unlike the relative gap, AEC has time units, facilitating a more intuitive interpretation of the level of equilibrium. In the Philadelphia regional network, Boyce et al. *(16)* found that link flows stabilized once the average excess cost was less than roughly 0.09 seconds (that is, when an average traveler's actual travel time is within a tenth of a second of the shortest path travel time). However, the current study uses aggregate measures of travel distance and time, rather than the values on individual links, and the former values may converge at a different rate than the latter. Figure 8 shows the convergence of these metrics for the three designs; similar plots for ADT are not shown for brevity. Both aggregate measures converge quickly, and achieve values

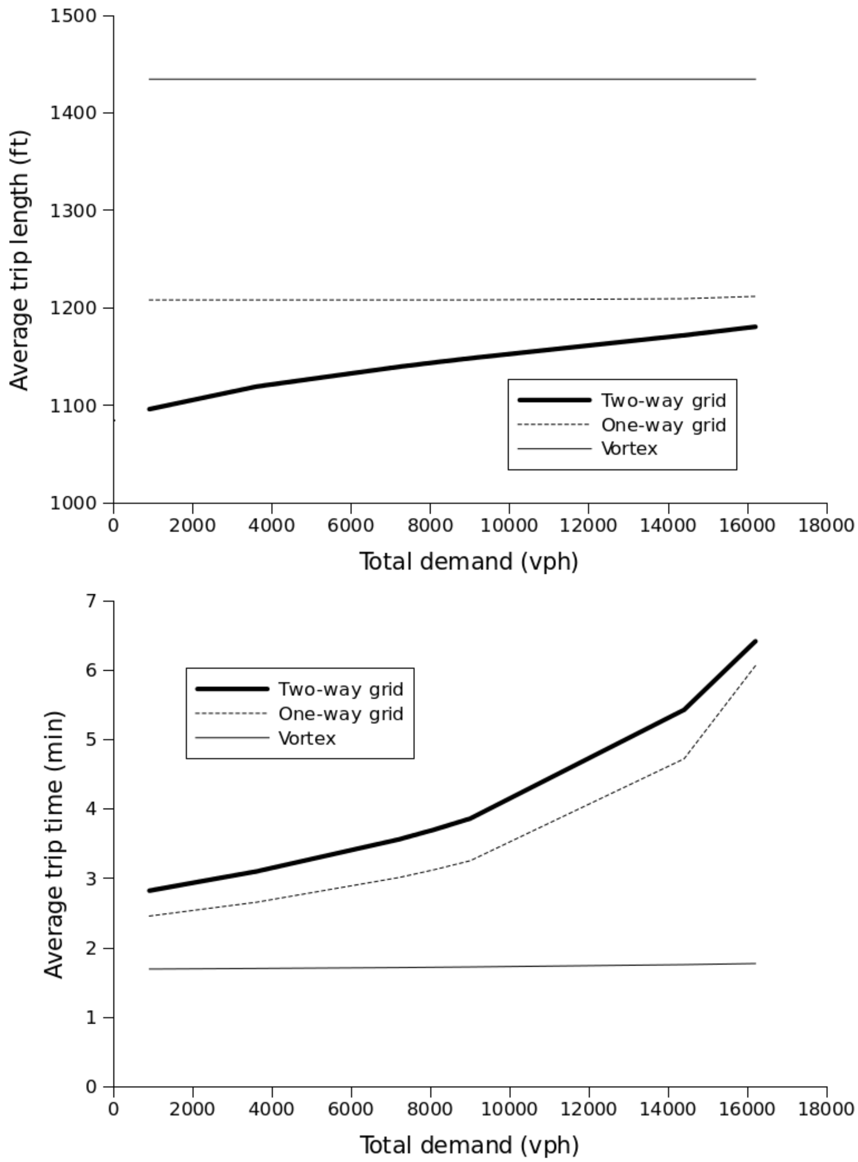

FIGURE 6 ADT and ATT as demand varies.

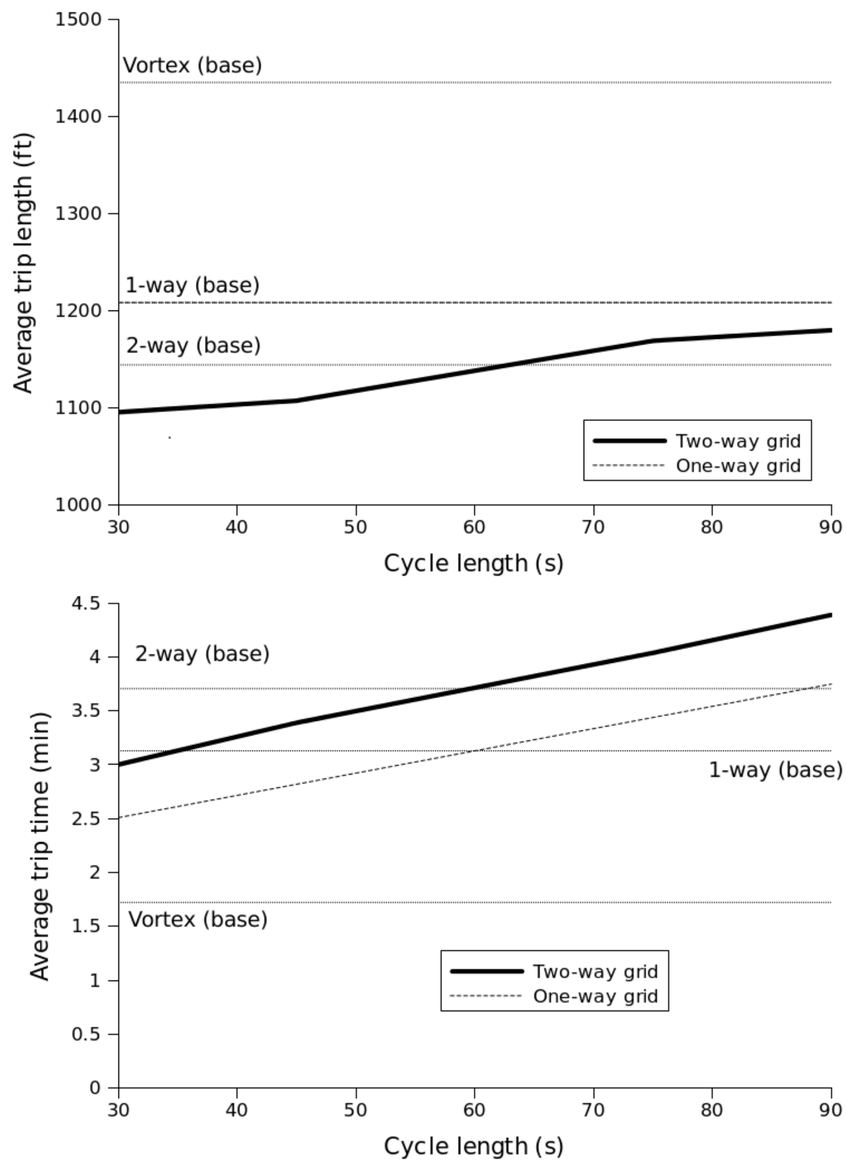

FIGURE 7 ADT and ATT as cycle length varies.

within 1% of those reported above even when the gap is greater than 1–2 seconds. Therefore, for the purposes of this paper, this algorithm appears to function well enough even though a different technique may be needed for larger networks.

5 STABILITY AT HIGH DEMAND LEVELS

A more surprising result occurs when demand is increased even further than the range initially tested. Figure 9 shows that the trends described in the previous section break down dramatically when the total demand is approximately 31,000 vph (corresponding to a per-block rate of one vehicle every nine seconds during the analysis hour). At a total demand of 30,500, the vortex-based design has an average travel time of 6 minutes per vehicle; but at a total demand of 31,000 the average travel time increases to 146 minutes. The average travel distance also increases rapidly at this point, although not nearly as quickly (ATD roughly doubles, while ATT increases more than twentyfold), This section attempts to explain this phenomenon and its implications.

Fundamentally, this effect can be traced to the combination of route switching and gap acceptance-based delay functions in the vortex network. The delay functions create massive congestion externalities, and the resulting network state is essentially the prisoner's dilemma writ large. Figure 10 shows a portion of a vortex network, drawn in the primal space for clarity. Consider a traveler beginning a trip at arc X, and wishing to travel to Y. At relatively low levels of congestion, the traveler will drive clockwise around block A, then proceed on vortex I to the destination. This traveler is delayed on link Z, and must wait for gaps in the traffic on vortex I. As the overall travel demand increases, the delay associated with this wait increases, until it becomes faster to drive an additional block further — rather than traveling clockwise around block A, the traveler will instead drive another block on vortex II, and ultimately turn onto vortex I by traveling clockwise around block D. Note that doing so further increases the delay for travelers on link Z, because this driver essentially traveled further to gain priority over others on link Z.

However, this route switching has increased the volume on vortex II, and induces still drivers at block C to switch routes in a similar way, traveling counterclockwise around block. In this way, a single vehicle switching routes can cause a cascade of route switching across the network, with each driver attempting to save time by driving slightly further out of the way to gain priority over other vehicles. However, each such effort increases the delay for other vehicles and induces still further switching, in the same way that players in the prisoner's dilemma make choices that slightly increase their utility at great expense to the other player.

Because this phenomenon is that it is fundamentally based on route choice behavior, simply evaluating these designs using a microsimulation software without large-scale route choice could not reveal this possibility — a predetermined set of routes will likely perform better than an equilibrated set. Figure 11 shows this by plotting ATD and ATT over early iterations of the method of successive averages. The initial assignment, based on free-flow conditions, has a somewhat higher ATD value than at lower demand levels, but not dramatically so, and in line with the trends established at lower demand levels in Figure 6. However, this solution is not an equilibrium, and the successive iterations show shifts towards shorter paths. As this happens, the ATD and ATT values increase very rapidly in the initial iterations, before stabilizing and then reducing slightly to their final values as the algorithm converges.

This phenomenon mirrors several effects which have been noted in the dynamic traffic assignment literature. Daganzo *(17)* presents a similar example in which vehicles divert to gain priority

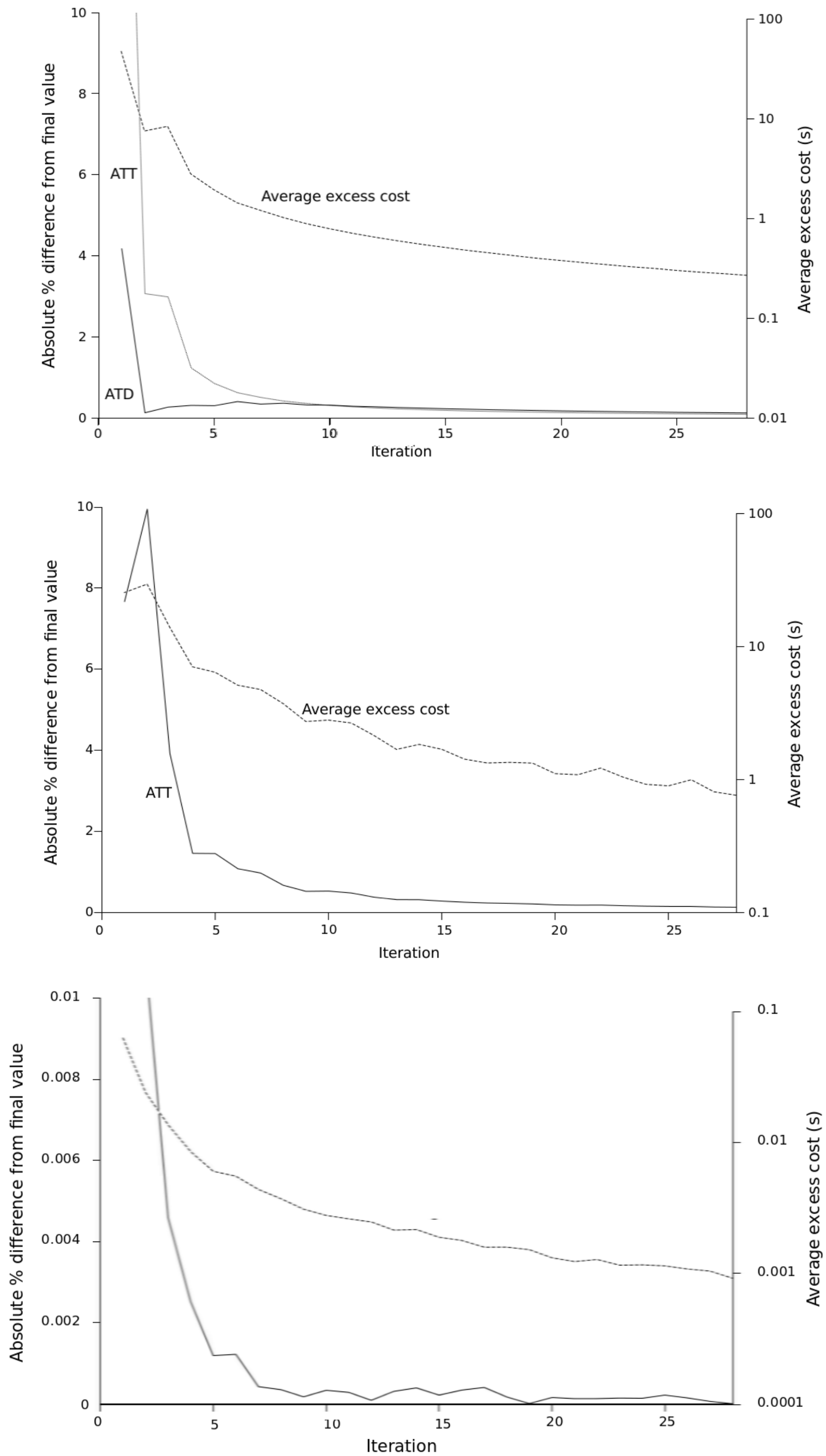

FIGURE 8 Convergence of aggregate measures and average excess cost for two-way grid (top), one-way grid (middle), and vortex (bottom) designs.

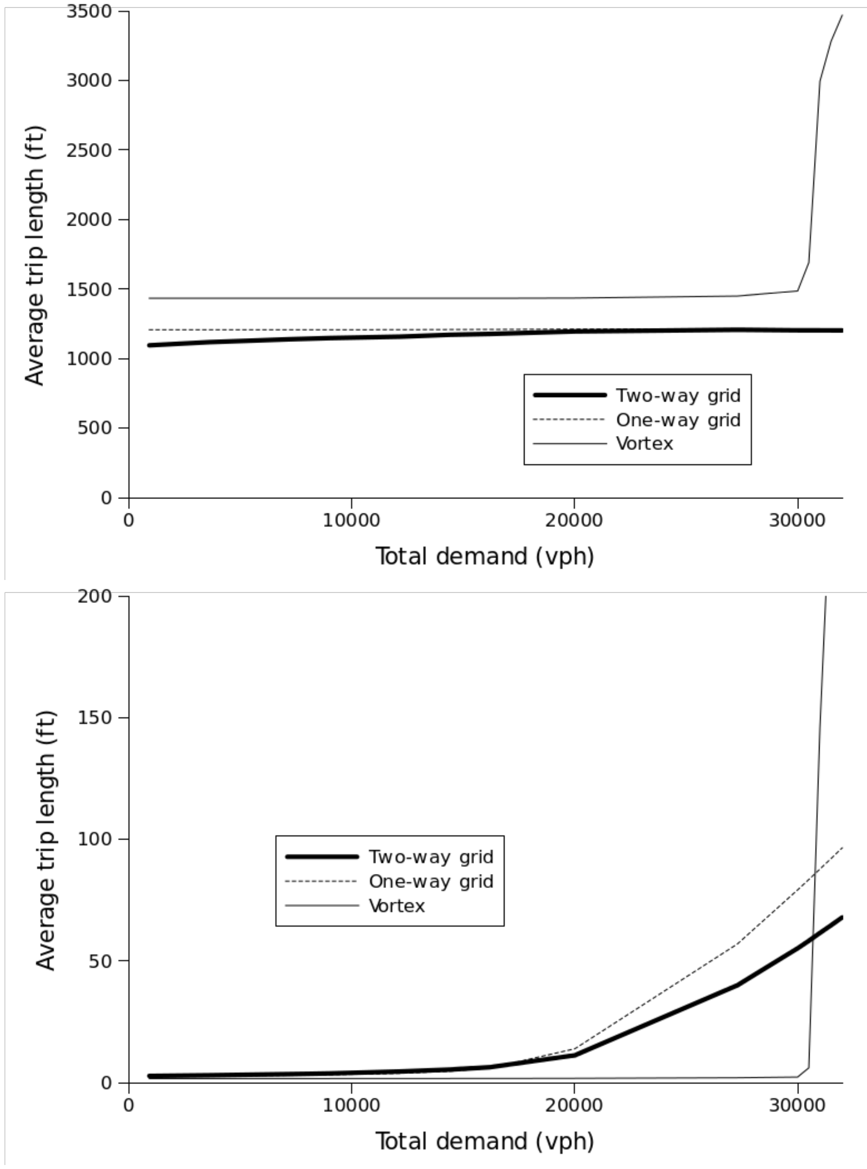

FIGURE 9 ADT and ATT as demand varies.

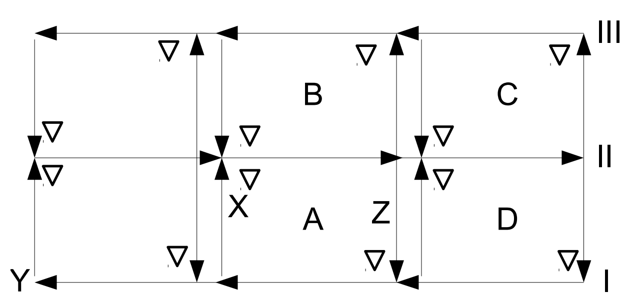

FIGURE 10 Diagram to demonstrate instability at high congestion.

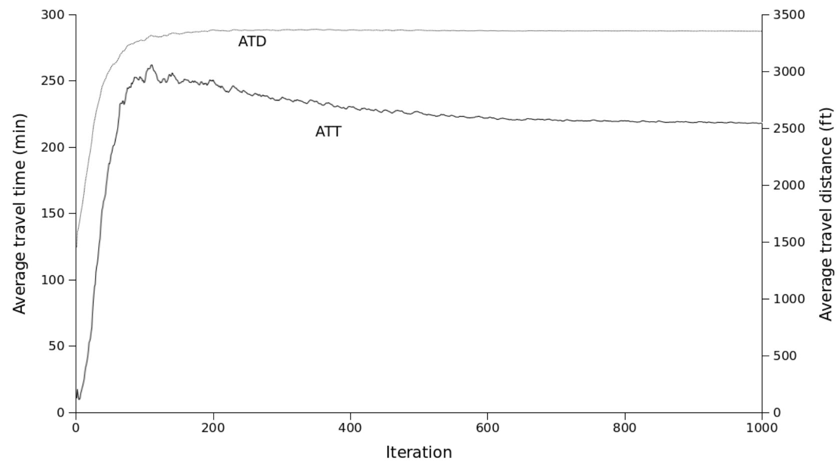

FIGURE 11 ADT and ATT increase rapidly in the initial iterations.

over those in a queue, and thereby produce an inefficient network condition. At high levels of congestion, vortex networks allow this to happen at virtually every block, resulting in the conditions described above. This system also presents the preconditions needed for "gridlocking" behavior to occur, as described in Daganzo *(18)*: as demand increases, not only are more vehicles queued at each yield junction, but the number of acceptable gaps decreases as well, and the increase in demand leads to a decrease in service rate. This lays the groundwork for negative feedback cycles to arise.

More fundamentally, this phenomenon demonstrates that interactions between the delay functions and route choice can trigger feedback loops deteriorating network conditions. Smith *(19)* presents an example in which Webster's rule for signal timing is suboptimal when combined with route choice. In his example, an initial allocation of green time based on volumes triggers shifts towards the link with greater green time; reapplying Webster's rule allocates still more green time to that approach, leading to even more route shifting towards that approach; and so on until all demand is using a single approach and the network capacity is inefficiently used. Likewise, in vortex networks, the asymmetry of the gap acceptance formulas effectively changes the delay functions on other links as vehicles reroute, leading to a similar feedback process.

As a remark, this phenomenon may also reflect the breakdown of gap acceptance formulas at high levels of congestion. The gap acceptance formulas used in this paper reflect perfect obedience on behalf of drivers waiting at a yield. In practice, as congestion sets in, some drivers at the yield exhibit "gap forcing" behavior, and some drivers on the priority stream will voluntarily allow waiting drivers to merge, as at freeway onramps during traffic jams. Developing delay functions to represent this behavior presents an interesting challenge for future research, perhaps leading to a "hybrid" delay function based on pure gap-acceptance at low priority stream volumes, then transitioning towards another functional form as volumes increase.

6 CONCLUSION

This paper presented a network equilibrium-based approach for evaluating novel network design strategies based on eliminating crossing conflicts altogether. As a point of comparison, the novel design was compared with more traditional grids of one-way and two-way streets with signal control. Appropriate delay functions were developed, based on signal delay and gap acceptance concepts. At lower levels of congestion, the vortex-based designs indeed reduced average travel time, at the cost of an increase in average travel distance. However, at very high levels of congestion, the use of gap-acceptance formulas for the vortex control generated a feedback loop, in which travelers chose longer routes to gain priority at merges, and such choices induced other travelers to do the same. This resulted in an abrupt worsening of system conditions. This latter phenomenon highlights the importance of including the route choice dimension in network evaluation, since an initial assignment of paths based on free-flow times results in much better network performance than what arises once equilibrium and route switching are accounted for.

A natural extension of this paper is to develop methods for designing and optimizing networks using these topologies, as is developing other delay functions to represent gap acceptance behavior at higher levels of flow or more advanced signal behavior such as progression or actuation. Furthermore, the evaluation framework presented in this paper may spur further research into the efficacy of vortex-based patterns or hybrid vortex/signal networks, such as those proposed in *(1)*. The use of a dynamic traffic flow model would provide additional insights into the behavior of such networks, but would require careful attention to the more complex nature of dynamic equilibrium. The issue of multiple equilibria can be studied more comprehensively. All of these are valuable subjects for future research.

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