Linear Programming

Graphical method

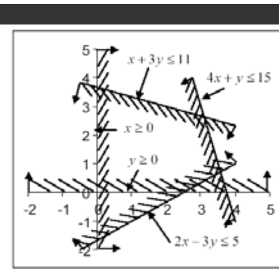
Objectives

- To visualize the optimization procedure explicitly
- To understand the different terminologies associated with the solution of LPP
- To discuss an example with two decision variables

Example

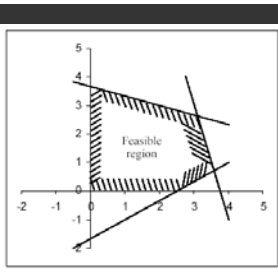
Maximize	$Z = 6x + 5y$	
subject to	$2x - 3y \leq 5$	(c-1)
	$x + 3y \leq 11$	(c-2)
	$4x + y \leq 15$	(c-3)
	$x, \ y \geq 0$	(c-4 & c-5)

Graphical method: Step - 1



Plot all the constraints one by one on a graph paper

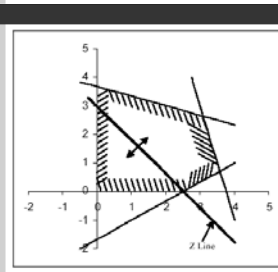
Graphical method: Step - 2



Identify the common region of all the constraints.

This is known as *'feasible region'*

Graphical method: Step - 3

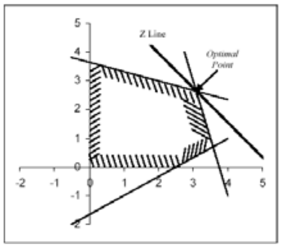


Plot the objective function, *Z*, assuming any constant, *k*, i.e.

$$6x + 5y = k$$

This is known as *'Z line'*, which can be shifted perpendicularly by changing the value of *k*.

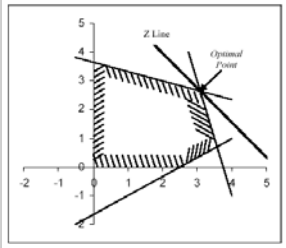
Graphical method: Step - 4



Notice that value of the objective function will be maximum when it passes through the intersection of $x+3y=11$ and $4x+y=15$ (straight lines associated with 2nd and 3rd constraints).

This is known as '*Optimal Point*'

Graphical method: Step - 5



Thus the *optimal point* of the present problem is

$$x^* = 3.091$$
$$y^* = 2.636$$

And the optimal solution is

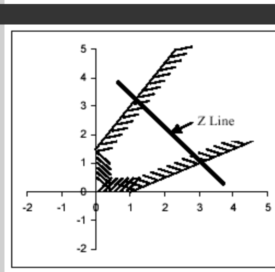
$$6x^* + 5y^* = 31.726$$

Different cases of optimal solution

A linear programming problem may have

1. A unique, finite solution (example already discussed)
2. An unbounded solution,
3. Multiple (or infinite) number of optimal solution,
4. Infeasible solution, and
5. A unique feasible point.

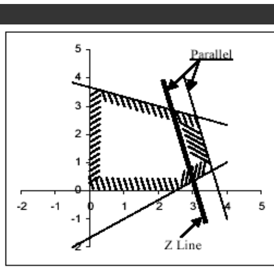
Unbounded solution: Graphical representation



Situation: If the feasible region is not bounded

Solution: It is possible that the value of the objective function goes on increasing without leaving the feasible region, i.e., unbounded solution

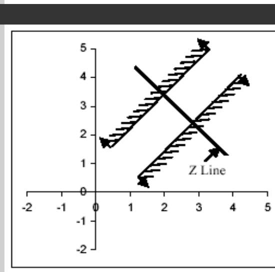
Multiple solutions: Graphical representation



Situation: *Z line* is parallel to any side of the feasible region

Solution: All the points lying on that side of the feasible region constitute optimal solutions

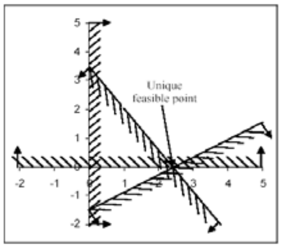
Infeasible solution: Graphical representation



Situation: Set of constraints does not form a feasible region at all due to inconsistency in the constraints

Solution: Optimal solution is not feasible

Unique feasible point: Graphical representation



Situation: Feasible region consist of a single point. Number of constraints should be at least equal to the number of decision variables

Solution: There is no need for optimization as there is only one feasible point