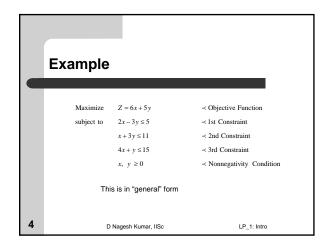
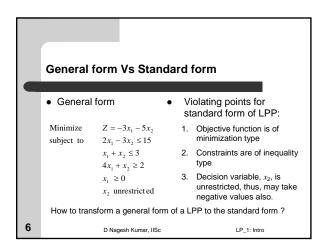


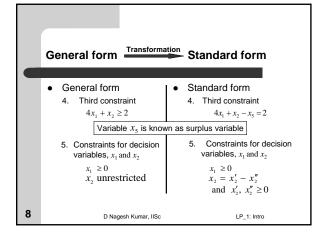
Linear Programming (LP) is the most useful optimization technique Objective function and constraints are the 'linear' functions of 'nonnegative' decision variables Thus, the conditions of LP problems are Objective function must be a linear function of decision variables Constraints should be linear function of decision variables All the decision variables must be nonnegative B Nagesh Kumar, IISc LP_1: Intro



Standard form of LP problems Standard form of LP problems must have following three characteristics: 1. Objective function should be of maximization type 2. All the constraints should be of equality type 3. All the decision variables should be nonnegative



General form Transformation Standard form Standard form General form 1. Objective function 1. Objective function Minimize $Z = -3x_1 - 5x_2$ Maximize $Z' = -Z = 3x_1 + 5x_2$ 2. First constraint 2. First constraint $2x_1 - 3x_2 + x_3 = 15$ $2x_1 - 3x_2 \le 15$ 3. Second constraint 3. Second constraint $x_1 + x_2 + x_4 = 3$ $x_1 + x_2 \le 3$ Variables x_3 and x_4 are known as slack variables 7 D Nagesh Kumar, IISc LP_1: Intro



Canonical form of LP Problems

- The 'objective function' and all the 'equality constraints' (standard form of LP problems) can be expressed in canonical
- Canonical form of LP problems is essential for simplex method (will be discussed later)
- Canonical form of a set of linear equations will be discussed next.

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Canonical form of a set of linear equations

Let us consider the following example of a set of linear equations

$$3x + 2y + z = 10 (A_0)$$

$$x - 2y + 3z = 6 (B_0)$$

$$2x + y - z = 1$$
 (C₀)

The system of equation will be transformed through 'Elementary Operations'.

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Elementary Operations

The following operations are known as elementary operations:

- 1. Any equation E_r can be replaced by kE_r , where k is a nonzero
- 2. Any equation E_r can be replaced by $E_r + kE_S$, where E_S is another equation of the system and k is as defined above.

Note: Transformed set of equations through elementary operations is equivalent to the original set of equations. Thus, solution of transformed set of equations is the solution of original set of equations too

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Transformation to Canonical form: An Example

Set of equation $(A_0, B_0 \text{ and } C_0)$ is transformed through elementary operations (shown inside bracket in the right side)

$$3x + 2y + z = 10$$

$$3x + 2y + z = 10$$
 \longrightarrow $x + \frac{2}{3}y + \frac{1}{3}z = \frac{10}{3}$

$$\left(A_{1} = \frac{1}{3}A_{0}\right)$$

$$x - 2y + 3z = 6 \qquad --$$

$$x-2y+3z=6$$
 \longrightarrow $0-\frac{8}{3}y+\frac{8}{3}z=\frac{8}{3}$ $(B_1=B_0-A_1)$

$$(B_1 = B_2 - A_1)$$

$$2x + y - z = 1$$
 $0 - \frac{1}{3}y - \frac{5}{3}z = -\frac{17}{3}$

$$0 - \frac{1}{3}y - \frac{5}{3}z = -\frac{17}{3}$$

$$(C_{1} = C_{2} - 2A_{1})$$

Note that variable x is eliminated from B_0 and C_0 equations to obtain B_1 and $C_1.$ Equation A_0 is known as pivotal equation.

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Transformation to Canonical form: Example contd.

Following similar procedure, y is eliminated from equation A_1 and C_1 considering B_1 as pivotal equation:

$$x+0+z=4$$
 $\left(A_2 = A_1 - \frac{2}{3}B_2\right)$
 $0+y-z=-1$ $\left(B_2 = -\frac{3}{8}B_1\right)$
 $0+0-2z=-6$ $\left(C_3 = C_1 + \frac{1}{2}B_3\right)$

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Transformation to Canonical form: Example contd.

Finally, z is eliminated form equation A_2 and B_2 considering C_2 as pivotal equation :

$$x+0+0=1$$
 $(A_3 = A_2 - C_3)$
 $0+y+0=2$ $(B_3 = B_2 + C_3)$
 $0+0+z=3$ $(C_3 = -\frac{1}{2}C_2)$

Note: Pivotal equation is transformed first and using the transformed pivotal equation other equations in the system are transformed.

The set of equations $(A_3,B_3$ and $C_3)$ is said to be in *Canonical form* which is equivalent to the original set of equations $(A_0,B_0$ and $C_0)$

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Pivotal Operation

Operation at each step to eliminate one variable at a time, from all equations except one, is known as *pivotal operation*.

Number of *pivotal operations* are same as the number of variables in the set of equations.

Three *pivotal operations* were carried out to obtain the canonical form of set of equations in last example having three variables.

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Transformation to Canonical form: Generalized procedure

Consider the following system of n equations with n variables

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$
 (E₁)

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$
 (E_2) \vdots

$$a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n$$
 (E_n)

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Transformation to Canonical form: Generalized procedure

Canonical form of above system of equations can be obtained by performing *n* pivotal operations

Variable x_i $(i = 1 \cdots n)$ is eliminated from all equations except j th equation for which a_{ij} is nonzero.

General procedure for one pivotal operation consists of following two steps,

1. Divide j^{th} equation by a_{ji} . Let us designate it as (E'_j) , i.e., $E'_j = \frac{E_j}{a_{ji}}$

2. Subtract a_{ki} times of (E'_j) equation from

 k^{th} equation $\left(k=1,2,\cdots j-1,j+1,\cdots,n\right)$, i.e., $E_k-a_{ki}E_j'$

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Transformation to Canonical form: Generalized procedure

After repeating above steps for all the variables in the system of equations, the canonical form will be obtained as follows:

$$1x_1 + 0x_2 + \dots + 0x_n = b_1'' \tag{E_1^c}$$

It is obvious that solution of above set of equation such as $x_i = b_i^n$ is the solution of original set of equations also.

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Transformation to Canonical form: More general case

Consider more general case for which the system of equations has m equation with n variables $(n \ge m)$

It is possible to transform the set of equations to an equivalent canonical form from which at least one solution can be easily deduced

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Transformation to Canonical form: More general case

By performing n pivotal operations for any m variables (say, $x_1, x_2, \cdots x_m$) called pivotal variables) the system of equations is reduced to canonical form as follows

$$1x_1 + 0x_2 + \dots + 0x_m + a_{1,m+1}'' x_{m+1} + \dots + a_{1n}'' x_n = b_1''$$
 (E₁)

Variables, x_{m+1},\cdots,x_n , of above set of equations is known as *nonpivotal variables* or independent variables.

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Basic variable, Nonbasic variable, Basic solution, Basic feasible solution

One solution that can be obtained from the above set of equations is

$$x_i = b_i''$$
 for $i = 1, \dots, m$
 $x_i = 0$ for $i = (m+1), \dots, n$

This solution is known as basic solution.

Pivotal variables, $x_1, x_2, \dots x_m$, are also known as *basic variables*.

Nonpivotal variables, x_{m+1}, \dots, x_n , are known as *nonbasic variables*.

Basic solution is also known as basic feasible solution because it satisfies all the constraints as well as non-negativity criterion for all the variables

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