



## Systems Techniques in Water Resources & Environmental Engg

**Prof. D. Nagesh Kumar**  
Dept. of Civil Engg.  
Indian Institute of Science  
Bangalore – 560 012, India

URL: <http://www.civil.iisc.ernet.in/~nagesh>

1

D Nagesh Kumar, IISc

Optimization - Introduction: L1

## Systems Techniques in Water Resources & Environmental Engg (3:0)

### Syllabus:

Optimization Techniques - Constrained and Unconstrained optimization; Kuhn-Tucker conditions; Linear Programming (LP); Dynamic Programming (DP); Multi-objective optimization; Applications in Water Resources, Water allocation, Reservoir sizing, Multipurpose reservoir operation for hydropower, flood control and irrigation. Review of probability theory; Stochastic optimization - Chance constrained LP, Stochastic DP; Surface water quality control; Simulation - Reliability, Resiliency and Vulnerability of water resource systems.

### References:

1. *Water Resources Systems Planning and Analysis*  
Loucks, D.P, Stedinger, J.R and Haith,D.A, Prentice Hall, Englewood Cliffs, NJ, 1981.
2. *Hydrosystems Engineering and Management*  
Mays,L.W and Tung, Y-K,McGraw Hill, 1992.
3. *Water Resources Systems: Modelling Techniques and Analysis*  
Vedula, S. and Mujumdar, P. P., Tata-McGraw Hill, 2005.
4. *Multicriterion Analysis in Engineering and Management*  
K. Srinivasa Raju and D. Nagesh Kumar, PHI Ltd., New Delhi, 2010, pp.288

<http://www.civil.iisc.ernet.in/~nagesh/stwree.htm>

2

## Systems Techniques in Water Resources & Environmental Engg (3:0)

### NPTEL Material

Water Resources Systems Planning and Management - Web  
By D. Nagesh Kumar  
<http://www.nptel.iitm.ac.in/courses/105108081>

Water Resources Systems : Modeling Techniques and Analysis –  
Videos by Prof. P.P. Mujumdar  
<http://www.nptel.iitm.ac.in/courses/105108130/>

3

D Nagesh Kumar, IISc

Optimization - Introduction: L1

## Evaluation

- **Assignments - 5%**
- **Two Class Tests - 2x15%**
- **Seminar - 15%** (Term Paper + Seminar)
- **Final Test - 50%**
- **Attendance <80% - -5%**

4

D Nagesh Kumar, IISc

Optimization - Introduction: L1

## Introduction and Basic Concepts

Historical  
Development and  
Model Building

5

D Nagesh Kumar, IISc

Optimization - Introduction: L1

## Introduction

- Optimization : The act of obtaining the best result under the given circumstances.
- Design, construction and maintenance of engineering systems involve decision making both at the managerial and the technological level
- Goals of such decisions :
  - to minimize the effort required or
  - to maximize the desired benefit

6

D Nagesh Kumar, IISc

Optimization - Introduction: L1

## Introduction (contd.)

- Optimization : The process of finding the conditions that give the minimum or maximum value of a function, where the function represents the effort required or the desired benefit.

7

D Nagesh Kumar, IISc

Optimization - Introduction: L1

## Historical Development

- Existence of optimization methods can be traced to the days of Newton, Lagrange and Cauchy.
- Development of differential calculus methods of optimization was possible because of the contributions of Newton and Leibnitz to calculus.
- Foundations of calculus of variations, dealing with the minimizations of functions, were laid by Bernoulli, Euler, Lagrange and Weistrass

8

D Nagesh Kumar, IISc

Optimization - Introduction: L1

## Historical Development (contd.)

- The method of optimization for constrained problems, which involve the inclusion of unknown multipliers, became known by the name of its inventor, Lagrange.
- Cauchy made the first application of the steepest descent method to solve unconstrained optimization problems.

9

D Nagesh Kumar, IISc

Optimization - Introduction: L1

## Recent History

- High-speed digital computers made implementation of the complex optimization procedures possible and stimulated further research on newer methods.
- Massive literature on optimization techniques and emergence of several well defined new areas in optimization theory followed.

10

D Nagesh Kumar, IISc

Optimization - Introduction: L1

## Milestones

- Development of the simplex method by Dantzig in 1947 for linear programming problems.
- The enunciation of the principle of optimality in 1957 by Bellman for dynamic programming problems.
- Work by Kuhn and Tucker in 1951 on the necessary and sufficient conditions for the optimal solution of problems laid the foundation for later research in non-linear programming.

11

D Nagesh Kumar, IISc

Optimization - Introduction: L1

## Milestones (contd.)

- The contributions of Zoutendijk and Rosen to nonlinear programming during the early 1960s
- Work of Carroll and Fiacco and McCormick facilitated many difficult problems to be solved by using the well-known techniques of unconstrained optimization.
- Geometric programming was developed in the 1960s by Duffin, Zener, and Peterson.
- Gomory did pioneering work in integer programming. The most real world applications fall under this category of problems.
- Dantzig, Charnes and Cooper developed stochastic programming techniques.

12

D Nagesh Kumar, IISc

Optimization - Introduction: L1

## Milestones (contd.)

- The desire to optimize more than one objective or a goal while satisfying the physical limitations led to the development of multi-objective programming methods; Ex. **Goal programming**.
- The foundations of game theory were laid by von Neumann in 1928 ; applied to solve several mathematical, economic and military problems and more recently to engineering design problems.
- Simulated annealing, evolutionary algorithms including genetic algorithms and neural network methods represent a new class of mathematical programming techniques that have come into prominence during the last decade.

13

D Nagesh Kumar, IISc

Optimization - Introduction: L1

## Engineering applications of optimization

- Design of structural units in construction, machinery and in space vehicles.
- Maximizing benefit/ minimizing product costs in various manufacturing and construction processes.
- Optimal path finding in road networks/ freight handling processes.
- Optimal production planning, controlling and scheduling.
- Optimal allocation of resources or services among several activities to maximize the benefit.

14

D Nagesh Kumar, IISc

Optimization - Introduction: L1

## Art of Modeling : Model Building

- Development of an optimization model can be divided into five major phases.
  - Collection of data
  - Problem definition and formulation
  - Model development
  - Model validation and evaluation of performance
  - Model application and interpretation of results

15

D Nagesh Kumar, IISc

Optimization - Introduction: L1

## Data collection

- **Data collection**
  - may be time consuming but is the fundamental basis of the model-building process
  - extremely important phase of the model-building process
  - the availability and accuracy of data can have considerable effect on the accuracy of the model and on the ability to evaluate the model.

16

D Nagesh Kumar, IISc

Optimization - Introduction: L1

## Problem Definition

- Problem definition and formulation, steps involved:
  - identification of the decision variables;
  - **formulation of the model objective(s)**;
  - formulation of the model constraints.
- In performing these steps one must consider the following.
  - **Identify the important elements that the problem consists of.**
  - Determine the number of independent variables, number of equations required to describe the system and the number of unknown parameters.
  - **Evaluate the structure and complexity of the model**
  - Select the degree of accuracy required of the model

17

D Nagesh Kumar, IISc

Optimization - Introduction: L1

## Model development

- **Model development** includes:
  - the mathematical description,
  - parameter estimation,
  - input development, and
  - software development
- The model development phase is an iterative process that may require returning to the model definition and formulation phase.

18

D Nagesh Kumar, IISc

Optimization - Introduction: L1

## Model Validation and Evaluation

- This phase is checking the model as a whole.
- **Model validation** consists of validation of the assumptions and parameters of the model.
- The performance of the model is to be evaluated using standard performance measures such as Root mean squared error and  $R^2$  values.
- Sensitivity analysis to test the model inputs and parameters.
- This phase also is an iterative process and may require returning to the model definition and formulation phase.
- One important aspect of this process is that in most cases data used in the formulation process should be different from that used in validation.

19

D Nagesh Kumar, IISc

Optimization - Introduction: L1

## Modeling Techniques

- Different modeling techniques are developed to meet the requirement of different type of optimization problems. Major categories of modeling approaches are:
  - classical optimization techniques
  - linear programming
  - nonlinear programming
  - geometric programming
  - dynamic programming
  - integer programming
  - stochastic programming
  - evolutionary algorithms etc.
- These approaches will be discussed in the subsequent lectures.

20

D Nagesh Kumar, IISc

Optimization - Introduction: L1

## Introduction and Basic Concepts

### Optimization Problem and Model Formulation

21

D Nagesh Kumar, IISc

Optimization - Introduction: L1

## Introduction - Preliminaries

- Basic components of an optimization problem :
  - An **objective function** expresses the main aim of the model which is either to be minimized or maximized.
  - A set of **unknowns** or **variables** which control the value of the objective function.
  - A set of **constraints** that allow the unknowns to take on certain values but exclude others.

22

D Nagesh Kumar, IISc

Optimization - Introduction: L1

## Introduction (contd.)

- The optimization problem is then to:
  - find values of the **variables** that minimize or maximize the **objective function** while satisfying the **constraints**.

23

D Nagesh Kumar, IISc

Optimization - Introduction: L1

## Objective Function

- As already defined the objective function is the mathematical function one wants to maximize or minimize, subject to certain constraints. Many optimization problems have a single objective function (when they don't they can often be reformulated so that they do).

The two interesting exceptions are:

- **No objective function.** The user does not particularly want to optimize anything so there is no reason to define an objective function. Usually called a **feasibility problem**.
- **Multiple objective functions.** In practice, problems with multiple objectives are reformulated as single-objective problems by either forming a weighted combination of the different objectives or by treating some of the objectives as constraints.

24

D Nagesh Kumar, IISc

Optimization - Introduction: L1

## Statement of an optimization problem

$$\text{To find } \mathbf{X} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \text{ which maximizes } f(\mathbf{X})$$

Subject to the constraints

$$\begin{aligned} g_i(\mathbf{X}) &\leq 0, & i = 1, 2, \dots, m \\ h_j(\mathbf{X}) &= 0, & j = 1, 2, \dots, p \end{aligned}$$

25

D Nagesh Kumar, IISc

Optimization - Introduction: L1

## Statement of an optimization problem

where

- $\mathbf{X}$  is an  $n$ -dimensional vector called the design vector
- $f(\mathbf{X})$  is called the *objective function*, and
- $g_i(\mathbf{X})$  and  $h_j(\mathbf{X})$  are known as inequality and equality constraints, respectively.
- This type of problem is called a *constrained optimization problem*.
- Optimization problems can be defined without any constraints as well. Such problems are called *unconstrained optimization problems*.

26

D Nagesh Kumar, IISc

Optimization - Introduction: L1

## Objective Function Surface

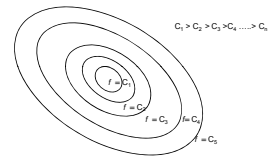
- If the locus of all points satisfying  $f(\mathbf{X}) = \text{a constant } c$  is considered, it can form a family of surfaces in the design space called the *objective function surfaces*.
- When drawn with the constraint surfaces as shown in the figure we can identify the optimum point (maxima).
- This is possible graphically only when the number of design variables is two.
- When we have three or more design variables because of complexity in the objective function surface we have to solve the problem as a mathematical problem and this visualization is not possible.

27

D Nagesh Kumar, IISc

Optimization - Introduction: L1

## Objective function surfaces to find the optimum point (maxima)

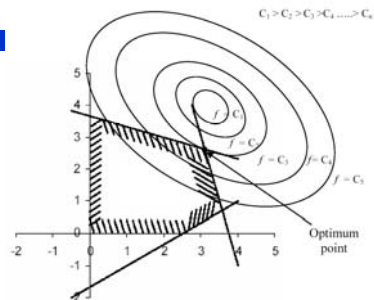


28

D Nagesh Kumar, IISc

Optimization - Introduction: L1

## Objective function surfaces with constraints to find the optimum point (maxima)



29

D Nagesh Kumar, IISc

Optimization - Introduction: L1

## Variables and Constraints

- **Variables**
  - These are essential. If there are no variables, we cannot define the objective function and the problem constraints.
- **Constraints**
  - Even though Constraints are not essential, it has been argued that almost all problems really do have constraints.
  - In many practical problems, one cannot choose the design variable arbitrarily. *Design constraints* are restrictions that must be satisfied to produce an acceptable design.

30

D Nagesh Kumar, IISc

Optimization - Introduction: L1

## Constraints (contd.)

- Constraints can be broadly classified as
  - Behavioral or Functional constraints: These represent limitations on the behavior and performance of the system.
  - Geometric or Side constraints: These represent physical limitations on design variables such as availability, fabricability, and transportability.

31

D Nagesh Kumar, IISc

Optimization - Introduction: L1

## Constraint Surfaces

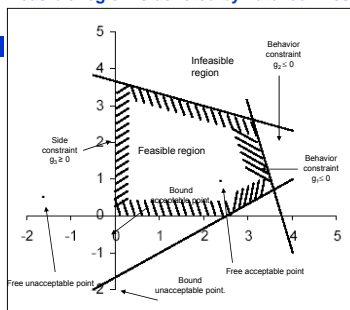
- Consider the optimization problem presented earlier with only inequality constraints  $g_i(\mathbf{X})$ . The set of values of  $\mathbf{X}$  that satisfy the equation  $g_i(\mathbf{X}) = 0$  forms a boundary surface in the design space called a *constraint surface*.
- The constraint surface divides the design space into two regions: one with  $g_i(\mathbf{X}) < 0$  (feasible region) and the other in which  $g_i(\mathbf{X}) > 0$  (infeasible region). The points lying on the hyper surface will satisfy  $g_i(\mathbf{X}) = 0$ .

32

D Nagesh Kumar, IISc

Optimization - Introduction: L1

A hypothetical two-dimensional design space where the feasible region is denoted by hatched lines



33

D Nagesh Kumar, IISc

Optimization - Introduction: L1

## Formulation of design problems as mathematical programming problems

- The following steps summarize the procedure used to formulate and solve mathematical programming problems.

- Analyze the process to identify the process variables and specific characteristics of interest i.e. make a list of all variables.
- Determine the criterion for optimization and specify the objective function in terms of the above variables together with coefficients.

34

D Nagesh Kumar, IISc

Optimization - Introduction: L1

- Develop via mathematical expressions a valid process model that relates the input-output variables of the process and associated coefficients.

- Include both equality and inequality constraints
- Use well known physical principles
- Identify the independent and dependent variables to get the number of degrees of freedom

- If the problem formulation is too large in scope:

- break it up into manageable parts or
- simplify the objective function and the model

- Apply a suitable optimization technique for mathematical statement of the problem.

- Examine the sensitivity of the result to changes in the coefficients in the problem and the assumptions.

35

D Nagesh Kumar, IISc

Optimization - Introduction: L1

## Maxima, Minima and Saddle Points

Maxima  
Minima  
Saddle Points

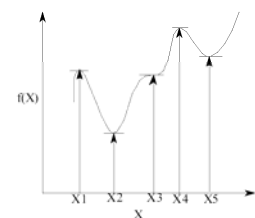
The first derivative is zero at these points

Local minima occur at  $x_2$  and  $x_4$   
 $f(x_2 - \Delta) > f(x_2) < f(x_2 + \Delta)$

Local maxima occur at  $x_1$  and  $x_3$   
 $f(x_1 - \Delta) < f(x_1) > f(x_1 + \Delta)$

Saddle point occurs at  $x_5$

**GLOBAL OPTIMUM**

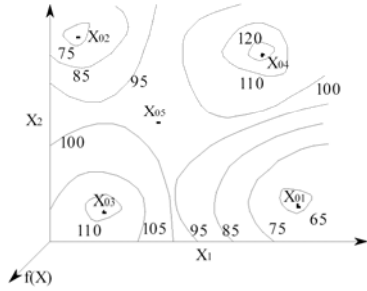


36

D Nagesh Kumar, IISc

Optimization - Introduction: L1

## Maxima, Minima and Saddle Points



37

D Nagesh Kumar, IISc

Optimization - Introduction: L1

# Thank You

38

D Nagesh Kumar, IISc

Optimization - Introduction: L1