Systems Techniques in Water Resources & Environmental Engg

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Systems Techniques in Water Resources & Environmental Engg (3:0)

Syllabus:
Optimization Techniques - Constrained and Unconstrained optimization;
Kuhn-Tucker conditions; Linear Programming (LP),
Dynamic Programming (DP); Multi-objective optimization;
Applications in Water Resources, Water allocation, Reservoir sizing,
Multipurpose reservoir operation for hydropower, flood control and irrigation.
Review of probability theory;
Stochastic optimization - Chance constrained LP, Stochastic DP;
Surface water quality control; Simulation - Reliability,
Resiliency and Vulnerability of water resource systems.

References:

1. Water Resources Systems Planning and Analysis
   Loucks, D.P, Stedinger, J.R and Haith,D.A, Prentice Hall, Englewood Cliffs, N.J, 1981.
2. Hydrosystems Engineering and Management
   Mays,L.W and Tung, Y-K,McGraw Hill, 1992.
3. Water Resources Systems: Modelling Techniques and Analysis
   Vedula, S. and Mujumdar, P. P., Tata-McGraw Hill, 2005.
4. Multicriterion Analysis in Engineering and Management
   K. Srinivasa Raju and D. Nagesh Kumar, PHI Ltd., New Delhi, 2010, pp.288

http://www.civil.iisc.ernet.in/~nagesh/stwree.htm

Systems Techniques in Water Resources & Environmental Engg (3:0)

NPTEL Material

Water Resources Systems Planning and Management - Web
By D. Nagesh Kumar
http://www.nptel.iitm.ac.in/courses/105108081

Water Resources Systems : Modeling Techniques and Analysis – Videos by Prof. P.P. Mujumdar
http://www.nptel.iitm.ac.in/courses/105108130/

Evaluation

- **Assignments - 5%**
- **Two Class Tests - 2x15%**
- **Seminar - 15%** **(Term Paper + Seminar)**
- **Final Test - 50%**
- **Attendance <80% - -5%**

Introduction and Basic Concepts

Historical Development and Model Building

Introduction

- Optimization : The act of obtaining the best result under the given circumstances.
- Design, construction and maintenance of engineering systems involve decision making both at the managerial and the technological level
- Goals of such decisions :
  - to minimize the effort required or
  - to maximize the desired benefit

Introduction (contd.)

- Optimization : The process of finding the conditions that give the minimum or maximum value of a function, where the function represents the effort required or the desired benefit.

Historical Development

- Existence of optimization methods can be traced to the days of Newton, Lagrange and Cauchy.
- Development of differential calculus methods of optimization was possible because of the contributions of Newton and Leibnitz to calculus.
- Foundations of calculus of variations, dealing with the minimizations of functions, were laid by Bernoulli, Euler, Lagrange and Weistrass

Historical Development (contd.)

- The method of optimization for constrained problems, which involve the inclusion of unknown multipliers, became known by the name of its inventor, Lagrange.
- Cauchy made the first application of the steepest descent method to solve unconstrained optimization problems.

Recent History

- High-speed digital computers made implementation of the complex optimization procedures possible and stimulated further research on newer methods.
- Massive literature on optimization techniques and emergence of several well defined new areas in optimization theory followed.

Milestones

- Development of the simplex method by Dantzig in 1947 for linear programming problems.
- The enunciation of the principle of optimality in 1957 by Bellman for dynamic programming problems.
- Work by Kuhn and Tucker in 1951 on the necessary and sufficient conditions for the optimal solution of problems laid the foundation for later research in non-linear programming.

Milestones (contd.)

- The contributions of Zoutendijk and Rosen to nonlinear programming during the early 1960s
- Work of Carroll and Fiacco and McCormick facilitated many difficult problems to be solved by using the well-known techniques of unconstrained optimization.
- Geometric programming was developed in the 1960s by Duffin, Zener, and Peterson.
- Gomory did pioneering work in integer programming. The most real world applications fall under this category of problems.
- Dantzig, Charnes and Cooper developed stochastic programming techniques.

Milestones (contd.)

- The desire to optimize more than one objective or a goal while satisfying the physical limitations led to the development of multi-objective programming methods; Ex. **Goal programming**.
- The foundations of game theory were laid by von Neumann in 1928 ; applied to solve several mathematical, economic and military problems and more recently to engineering design problems.
- Simulated annealing, evolutionary algorithms including genetic algorithms and neural network methods represent a new class of mathematical programming techniques that have come into prominence during the last decade.

Engineering applications of optimization

- Design of structural units in construction, machinery and in space vehicles.
- Maximizing benefit/ minimizing product costs in various manufacturing and construction processes.
- Optimal path finding in road networks/ freight handling processes.
- Optimal production planning, controlling and scheduling.
- Optimal allocation of resources or services among several activities to maximize the benefit.

Art of Modeling : Model Building

- Development of an optimization model can be divided into five major phases.
  - Collection of data
  - Problem definition and formulation
  - Model development
  - Model validation and evaluation of performance
  - Model application and interpretation of results

Data collection

- **Data collection**
  - may be time consuming but is the fundamental basis of the model-building process
  - extremely important phase of the model-building process
  - the availability and accuracy of data can have considerable effect on the accuracy of the model and on the ability to evaluate the model.

Problem Definition

- Problem definition and formulation, steps involved:
  - identification of the decision variables;
  - formulation of the model objective(s);
  - formulation of the model constraints.
- In performing these steps one must consider the following.
  - Identify the important elements that the problem consists of.
  - Determine the number of independent variables, number of equations required to describe the system and the number of unknown parameters.
  - Evaluate the structure and complexity of the model
  - Select the degree of accuracy required of the model

Model development

- **Model development** includes:
  - the mathematical description,
  - parameter estimation,
  - input development, and
  - software development
- The model development phase is an iterative process that may require returning to the model definition and formulation phase.

Model Validation and Evaluation

- This phase is checking the model as a whole.
- **Model validation** consists of validation of the assumptions and parameters of the model.
- The performance of the model is to be evaluated using standard performance measures such as Root mean squared error and R^2 values.
- Sensitivity analysis to test the model inputs and parameters.
- This phase also is an iterative process and may require returning to the model definition and formulation phase.
- One important aspect of this process is that in most cases data used in the formulation process should be different from that used in validation.

Modeling Techniques

- Different modeling techniques are developed to meet the requirement of different type of optimization problems. Major categories of modeling approaches are:
  - classical optimization techniques
  - linear programming
  - nonlinear programming
  - geometric programming
  - dynamic programming
  - integer programming
  - stochastic programming
  - evolutionary algorithms etc.
- These approaches will be discussed in the subsequent lectures.

Introduction and Basic Concepts

Optimization Problem and Model Formulation

Introduction - Preliminaries

- Basic components of an optimization problem :
  - An **objective function** expresses the main aim of the model which is either to be minimized or maximized.
  - A set of **unknowns** or **variables** which control the value of the objective function.
  - A set of **constraints** that allow the unknowns to take on certain values but exclude others.

Introduction (contd.)

- The optimization problem is then to:
  - **find values of the *variables* that minimize or maximize the *objective function* while satisfying the *constraints*.**

Objective Function

- As already defined the objective function is the mathematical function one wants to maximize or minimize, subject to certain constraints. Many optimization problems have a single objective function (when they don't they can often be reformulated so that they do).

The two interesting exceptions are:

- ***No objective function***. The user does not particularly want to optimize anything so there is no reason to define an objective function. Usually called a *feasibility problem*.
- ***Multiple objective functions***. In practice, problems with multiple objectives are reformulated as single-objective problems by either forming a weighted combination of the different objectives or by treating some of the objectives as constraints.

Statement of an optimization problem

To find $\mathbf{X} = \begin{pmatrix} x_1 \\ x_2 \\ . \\ . \\ x_n \end{pmatrix}$ *which maximizes* $f(\mathbf{X})$

Subject to the constraints

$g_i(\mathbf{X}) <= 0 , \quad i = 1, 2, \ldots, m$

$l_j(\mathbf{X}) = 0 , \quad j = 1, 2, \ldots, p$

Statement of an optimization problem

where

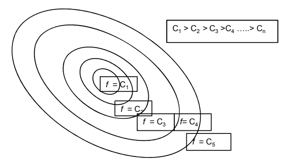
- **X** is an *n*-dimensional vector called the design vector
- *f*(**X**) is called the *objective function*, and
- $g_i(\mathbf{X})$ and $l_j(\mathbf{X})$ are known as inequality and equality constraints, respectively.

- This type of problem is called a *constrained optimization problem*.
- Optimization problems can be defined without any constraints as well. Such problems are called *unconstrained optimization problems.*

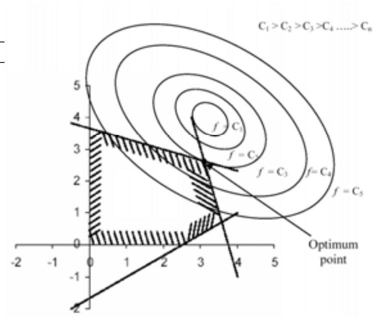
Objective Function Surface

- If the locus of all points satisfying ***f*(X) = a constant c** is considered, it can form a family of surfaces in the design space called the ***objective function surfaces.***
- When drawn with the constraint surfaces as shown in the figure we can identify the optimum point (maxima).
- This is possible graphically only when the number of design variables is two.
- When we have three or more design variables because of complexity in the objective function surface we have to solve the problem as a mathematical problem and this visualization is not possible.

Objective function surfaces to find the optimum point (maxima)



Objective function surfaces with constraints to find the optimum point (maxima)



Variables and Constraints

- Variables
  - These are essential. If there are no variables, we cannot define the objective function and the problem constraints.
- Constraints
  - Even though Constraints are not essential, it has been argued that almost all problems really do have constraints.
  - In many practical problems, one cannot choose the design variable arbitrarily. *Design constraints* are restrictions that must be satisfied to produce an acceptable design.

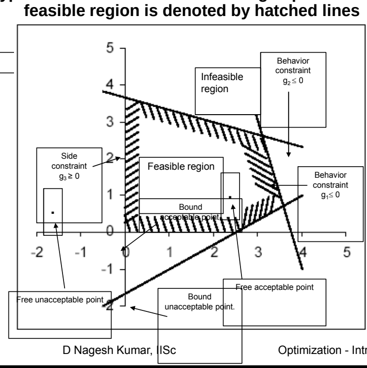
Constraints (contd.)

- Constraints can be broadly classified as
  - Behavioral or Functional constraints: These represent limitations on the behavior and performance of the system.
  - Geometric or Side constraints: These represent physical limitations on design variables such as availability, fabricability, and transportability.

Constraint Surfaces

- Consider the optimization problem presented earlier with only inequality constraints $g_i(\mathbf{X})$. The set of values of **X** that satisfy the equation $g_i(\mathbf{X})$ forms a boundary surface in the design space called a *constraint surface.*
- The constraint surface divides the design space into two regions: one with $g_i(\mathbf{X}) < 0$ (feasible region) and the other in which $g_i(\mathbf{X}) > 0$ (infeasible region). The points lying on the hyper surface will satisfy $g_i(\mathbf{X}) = 0$.

A hypothetical two-dimensional design space where the feasible region is denoted by hatched lines



Formulation of design problems as mathematical programming problems

- **The following steps summarize the procedure used to formulate and solve mathematical programming problems.**
  1. Analyze the process to identify the process variables and specific characteristics of interest i.e. make a list of all variables.
  2. Determine the criterion for optimization and specify the objective function in terms of the above variables together with coefficients.

3. Develop via mathematical expressions a valid process model that relates the input-output variables of the process and associated coefficients.
   a) Include both equality and inequality constraints
   b) Use well known physical principles
   c) Identify the independent and dependent variables to get the number of degrees of freedom
3. If the problem formulation is too large in scope:
   a) break it up into manageable parts or
   b) simplify the objective function and the model
4. Apply a suitable optimization technique for mathematical statement of the problem.
5. Examine the sensitivity of the result to changes in the coefficients in the problem and the assumptions.

Maxima, Minima and Saddle Points

Maxima
Minima
Saddle Points

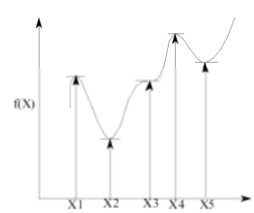
The first derivative is zero at these points

Local minima occur at x_2 and x_5
$f(x_2 - \Delta) > f(x_2) < f(x_2 + \Delta)$

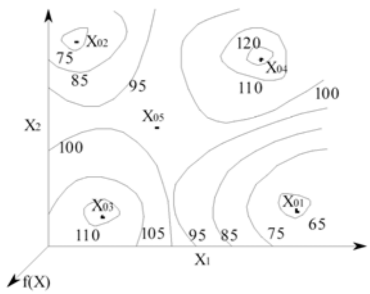
Local maxima occur at x_1 and x_4
$f(x_1 - \Delta) < f(x_1) > f(x_1 + \Delta)$

Saddle point occurs at x_3

GLOBAL OPTIMUM



Maxima, Minima and Saddle Points



Thank You