APPLICATION OF MULTI-OBJECTIVE FUZZY AND STOCHASTIC LINEAR PROGRAMMING TO SRI RAM SAGAR IRRIGATION PLANNING PROJECT OF ANDHRA PRADESH

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ABSTRACT

Fuzzy Linear Programming (FLP) is developed for the evaluation of management strategies for a case study of Sri Ram Sagar Project, Andhra Pradesh, India. Three conflicting objectives net benefits, crop production and labour employment are considered in the irrigation planning scenario. The present paper demonstrates how vagueness and imprecision in the objective function values can be quantified by membership functions in a fuzzy multiobjective framework. Uncertainty in the inflows is considered by stochastic programming. Analysis of results indicated that net benefits, crop production and labour employment in FLP are deviated by 2.38%, 9.6% and 7.22% as compared to ideal values in the crisp Linear Programming (LP) model. Comparison of results indicated that the methodology can be extended to other similar situations.

1 INTRODUCTION

Increasing demands for agricultural products with limited water resources lead to irrigation planning and management problems. In addition, the conflicting objectives of individual monetary benefits and social benefits, inevitability of uneconomical crops and providing employment to labour make the problems rather more complex. For efficient and scientific solutions of these problems ground water is also to be optimally extracted and combined with surface water to meet the requirements. At the same time it is necessary that the water quality standards are not seriously affected. Even at a marginal reduction of net benefits, labour deployment with increased cropped area could be a better alternative for the society. On the other hand, uncertainty makes irrigation planning problems more complex in the form of unexpected droughts and floods, uncertainty in price of crops, uncertainty in yields, nonavailability of labour at right time, variation of inflows from year to year.

Fuzzy set theory is identified as an alternative approach to handle such vagueness of the planning objectives and imprecision involved in the parameter values since deterministic approaches are not sufficient to model such complex situations. Chang et al. (1997) explained the advantage of fuzzy multiobjective optimization over deterministic approach as 1) fuzzy uncertainties embedded in the model parameters can be directly reflected and communicated into the optimization process 2) the variation or vagueness of the decision maker's aspiration level in the model can be incorporated and there by generate a more confident solution set for decision maker 3) regardless of the orientation of decision maker's aspiration level (maximization or minimization), each objective or goal have its own independent membership function and different aspiration levels.

The present study considered above aspects in the multiobjective Fuzzy Linear Programming (FLP) frame work by incorporating three objectives net benefits, crop production and labour employment for selection of the compromise irrigation plan. The present study deviates from previous studies by considering fuzziness in the objective functions, uncertainty in the inflows through stochastic programming and multiple objectives in the analysis.

2 MULTIOBJECTIVE FUZZY LINEAR PROGRAMMING

Fuzzy Linear Programming problem associates fuzzy input data by fuzzy membership functions. Fuzzy Linear Programming model assumes that objectives and constraints in an imprecise and uncertain situation can be represented by fuzzy sets. The fuzzy objective function can be maximized or minimized. In Fuzzy Linear Programming the fuzziness of available resources is characterised by the membership function over the tolerance range. In the present study objective functions are considered as fuzzy sets and inflows are considered in the form of chance constraints. In conventional LP, the problem is defined as follows (Zimmermann, 1996) :

Maximize $Z = C X$ (1)

Subject to $A X \leq B$ (2)

$X \geq 0$ (3)

In the Fuzzy Linear Programming the problem can be restated as

Find X such that

$C X \leq Z$ (4)

$A X \leq B$ (5)

and $X \geq 0$ (6)

The membership function of the fuzzy set 'decision model' is

$$m_D(X) = \min_i \{m_i (X)\}; \text{ i = 1, 2, ,n} \quad (7)$$

$m_i (X)$ can be interpreted as the degree to which X fulfils the fuzzy inequality $C X \leq Z$ and n is the number of objective functions. In the planning scenario, decision maker is not interested in a fuzzy set but in crisp optimum solution, maximizing equation (7) becomes

$$\text{Max}_{X \geq 0} \; m_D(X) = \text{Max}_{X \geq 0} \min_i \{m_i (X)\} \quad (8)$$

Membership function $m_i (X)$ is represented as

$$m_i(X) \quad 0 \text{ for } Z \quad Z_L$$
$$= \frac{Z - Z_L}{Z_U - Z_L} \text{ for } Z_L \leq Z \leq Z_U \quad (9)$$
$$= 1 \text{ for } Z > Z_U$$

Z_U = Aspired level of objective
Z_L = Lowest acceptable level of objective

$m_i(X)$ reflects the degree of achievement. Value of $m_i(X)$ will be 1 for perfect achievement and 0 for no-achievement (worst achievement) of a given strategy and some intermediate values otherwise. The model can be transformed as follows:

$$\text{Max}_{X \geq 0} \; \min_i \frac{Z - Z_L}{Z_U - Z_L} \quad (10)$$

subject to

$A X \leq B$ (11)

$X \geq 0$ (12)

Introducing new variable λ , the FLP problem can be formulated as equivalent LP model.

Max λ
subjected to

$$\frac{Z - Z_L}{Z_U - Z_L} \geq l \quad (13)$$

for each objective function

$A X \leq B$ (14)

$0 \leq l \leq 1$ (15)

$X \geq 0$ (16)

In brief the FLP algorithm is divided into six steps:

1. Solve the problem as a Linear Programming problem by taking only one of the objectives at a time.
2. From the results of step 1, determine the corresponding values of every objective at each solution derived.
3. From step 2, best (Z_U) and worst (Z_L) values can be calculated.
4. Formulate the linear membership function.
5. Formulate the equivalent Linear Programming model for the fuzzy multiobjective problem.
6. Determine the compromise solution along with degree of truth (λ) .

3 IRRIGATION SYSTEM FOR STUDY

The above methodology is applied to the case study of Sri Ram Sagar Project (SRSP), Andhra Pradesh, India. The culturable command area (CCA) of the project is 178,100 ha. Map showing the location of the project is presented in Fig 1. Main crops grown in the command area are Paddy, Maize, Sorghum, Groundnut, Vegetables, Pulses, Chillies and Sugarcane. Mathematical modelling of the three conflicting objectives with the corresponding constraints are briefly explained below.

Mathematical modelling

Objective 1: Maximization of net benefits

The net benefits (*BEM*) from the irrigated as well as unirrigated area under different crops is obtained by subtracting the costs of surface water, ground water, fertilizer and labour from the gross revenue for different crops. Maximization of net benefits can be expressed as

$$\text{Max } BEM = \sum_{i=1}^{16} B_i\ A_i - P_{sw} \sum_{t=1}^{12} R_t - P_{gw} \sum_{t=1}^{12} GW_t - \sum_{f=1}^{3} \sum_{i=1}^{16} F_{fi}\ A_i\ P_f - P_l \sum_{t=1}^{12} \sum_{i=1}^{16} L_{it}\ A_i \quad (17)$$

in which i = Crop index [1=Paddy(s), 2=Maize(s), 3=Sorghum(s), 4=Groundnut(s), 5=Vegetables(s), 6=Pulses(s), 7=Paddy(srf), 8=Groundnut(srf), 9=Paddy(w), 10=Groundnut(w), 11=Pulses(w), 12=Maize(w), 13=Sorghum(w), 14=Vegetables(w), 15=Chillies(w), 16=Sugarcane(ts)]; s=Summer; w=Winter; ts=Two season; srf=Summer rainfed; t =Monthly index; f =Fertilizer index; A_i = Area of crop i (ha); B_i = Unit gross return from i th crop (Rs) ; P_{sw} = Unit surface water cost (Rs / Mm^3); R_t = Monthly canal water releases (Mm^3); P_{gw} =Unit ground water cost (Rs /Mm^3); GW_t = Monthly ground water requirement (Mm^3); F_{fi} = Quantity of fertilizer of type f for crop i (tons/ha); P_f = Unit cost of fertilizer type f (Rs); P_l = Unit wage rate (Rs); L_{it} = Labour-days required for each hectare of crop i in month t; Rs=Rupees in Indian currency.

Objective 2: Maximization of crop production

Crop production (*PRM*) of Cereals, Pulses and Groundnut are maximized for meeting the demands and can be expressed as

$$\text{Max } PRM = \sum_{i} Y_i\ A_i\ ;\ i = 1,2,3,4,6,7,8,9,10,11,12,13 \quad (18)$$

where Y_i =Yield of i th crop (tons/ha).

Objective 3: Maximization of labour employment

The total labour employed (*LAM*) under all the crops for the whole year is maximized to increase the level of their economic status and can be expressed as

$$\text{Max } LAM = \sum_{t=1}^{12} \sum_{i=1}^{16} L_{it}\ A_i \quad (19)$$

The above given objectives are subject to the following constraints:

a) Continuity equation

The monthly continuity equation for the reservoir storage (Mm^3) is expressed as

$$S_{t+1} = S_t + Q_t - EV_t - R_t - RDS_t - OSR_t \quad (20)$$

where S_{t+1} = End of month reservoir storage volume; Q_t = Monthly net inflow volume; EV_t = Monthly net evaporation volume; RDS_t = Downstream requirements; OSR_t = Spilled volume

By incorporating the stochasticity in the inflow terms, the above equation changes to

$$\Pr(S_{t+1} - S_t + EV_t + R_t + RDS_t + OSR_t = Q_t) \geq a \quad (21)$$

$$S_{t+1} - S_t + EV_t + R_t + RDS_t + OSR_t \leq q_t^a \quad (22)$$

where q_t^a is the inverse of the cumulative distribution function of inflows at dependable level α, **Pr** is the probability operator.

b) Crop land requirements

The total area allocated for different crops in a particular season should be less than or equal to the culturable command area (*CCA*).

$$\sum_i A_i \leq CCA;\ i = 1,2,3,4,5,6,7,8,15,16 \text{ for summer crops} \quad (23)$$

$$\sum_i A_i \leq CCA;\ i = 9,10,11,12,13,14,15,16 \text{ for winter crops} \quad (24)$$

Crops of two seasons, namely, Chillies and Sugarcane (indices 15 and 16) are included in both the equations because they occupy the land in both seasons.

c) Water requirements of crops

Monthly crop water requirements should not exceed the maximum available water from both surface and ground water sources.

$$\sum_{i=1}^{16} A_i \; CWR_{it} \;\le\; R_t + GW_t \qquad (25)$$

where CWR_{it} is crop water requirement for unit area of crop i in month t.

The other constraints which are incorporated in the model are canal capacity restrictions, minimum and maximum reservoir storages, crop diversification considerations, downstream water requirements, labour and fertilizer availability, water quality, ground water withdrawals etc.

4 RESULTS AND DISCUSSION

4.1 STOCHASTIC PROGRAMMING

The monthly inflows into the Sri Ram Sagar reservoir are assumed to follow the log-normal distribution. Twenty three years of historical inflow data is used to obtain the various dependability levels of inflows. In the present study 90% dependability level inflows are considered. These are 132.10, 372.88, 798.50, 812.70, 352.02, 56.9, 36.00 Mm^3 respectively from June to December. The inflows of other months are not significant and are neglected (Maji and Heady, 1980).

4.2 INDIVIDUAL OPTIMIZATION

Optimization of each individual objective (labour, production and benefits) is performed with a Linear Programming (LP) algorithm that gave the upper and lower bounds for the multiobjective analysis. Results are presented in Table1. In the irrigation planning model, there is no significant change in acreage of Groundnut, Vegetables, Pulses in summer season, Paddy, Pulses, Vegetables in winter season and Sugarcane for all the three planning objectives. Irrigation intensity in labour, production and benefits maximization cases are 152.34%, 142.13%, 101.96% respectively. Cropping intensity is 197.92%, 173.87%, 154.33%. Ideal and worst values are denoted with an asterisk and plus respectively. As can be seen, the three planning objectives conflict with one another. There is a need to develop a tradeoff relationship and to select the compromise alternative cropping plan and the corresponding water allocation policies in the multiobjective irrigation planning context to meet the chosen levels of satisfaction as would be demanded in the decision making process.

4.3 MULTIOBJECTIVE FUZZY LINEAR PROGRAMMING

Values of best (Z_U) and worst (Z_L) are substituted in the equation (13) which results

Max λ

subjected to

$$-\lambda + 1.69 \times 10^{-9}\, BI \ge 2.84 \qquad (26)$$
$$-\lambda + 4.472 \times 10^{-6}\, PM \ge 3.478 \qquad (27)$$
$$-\lambda + 9.031 \times 10^{-8}\, LM \ge 4.1753 \qquad (28)$$

and all the existing constraints.

Results of FLP are presented in Table 1. It is observed that compromise solution favours both Groundnut (srf) and Groundnut (w) with an acreage of 97590 ha, 21940 ha. The compromise solution yields benefits Rupees 1633 x 10^6, 0.7030 x 10^6 tons of production, 42.89 x 10^6 man-days with degree of truth(λ) 0.6851. On the other hand solution is almost consistent for Groundnut (s), Vegetables (s & w), Pulses (s & w) and Paddy (w). In Paddy (srf) case the acreage becomes half as compared to labour and production cases. Irrigation intensity is less than labour and production cases but more than benefits case by 8%. However, cropping intensity is higher than the production and benefits case and less than labour case. Significant change is observed for Chillies where there is reduction of 55250 ha (in labour maximization) to 8600 ha in case of FLP. It is observed that net benefits, crop production and labour employment in FLP are deviated by 2.38%, 9.6% and 7.22% as compared to ideal values in the crisp Linear Programming (LP) model.

5 CONCLUSIONS

Based on the analysis of the results of a real world irrigation planning problem of Sri Ram Sagar Project, Andhra Pradesh, India, the following conclusions are drawn.

1. Fuzzy Linear Programming (FLP) is simple and suitable tool for multiobjective problems compared to other methods.
2. The model can be extended to any number of objectives by incorporating only one additional constraint in the constraint set for each additional objective function.
3. The model can be extended to any situation not only irrigation planning but any field of engineering with little or no modifications.
4. Analysis of results indicated that net benefits, crop production and labour employment in FLP are deviated by 2.38%, 9.6% and 7.22% as compared to ideal values in the crisp Linear Programming (LP) model.

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Table 1. Ideal crop plans from the planning model and Fuzzy Linear Programming.

Crops and related parameters		Units	Solution for maximization of			
			Labour	Production	Benefits	FLP
1. Paddy	(s)	1000ha	2.000	2.000	62.93	24.46
2. Maize	(s)	1000ha	5.000	54.65	5.000	5.00
3. Sorghum	(s)	1000ha	22.87	50.00	1.900	1.90
4. Groundnut	(s)	1000ha	1.500	1.500	1.500	1.50
5 Vegetables	(s)	1000ha	2.000	2.000	2.100	2.00
6. Pulses	(s)	1000ha	4.200	4.200	4.200	4.20
7. Paddy	(srf)	1000ha	51.20	51.20	0.000	28.73
8. Groundnut	(srf)	1000ha	29.97	5.340	93.26	97.59
9. Paddy	(w)	1000ha	14.70	14.70	14.70	14.70
10.Groundnut	(w)	1000ha	5.700	5.700	15.81	21.94
11. Pulses	(w)	1000ha	39.85	39.85	39.85	39.85
12. Maize	(w)	1000ha	13.00	22.33	13.00	13.00
13. Sorghum	(w)	1000ha	40.00	40.00	4.500	40.00
14. Vegetables	(w)	1000ha	1.800	1.800	1.700	1.800
15. Chillies	(ts)	1000ha	55.25	3.100	3.100	8.600
16 . Sugarcane	(ts)	1000ha	4.100	4.100	4.100	4.100
Irrigation Intensity		%	152.34	142.13	101.96	109.92
Cropping Intensity		%	197.92	173.87	154.33	180.85

Payoff matrix

Net Benefits(10^6 Rupees)	1418.6000	1084.0000^+	1672.9000^*	1633.0
Crop Prod (10^6 Tons)	0.5542^+	0.7778^*	0.6807	0.7030
Labour Util.(10^6 Man-Days)	46.2290^*	35.1570^+	40.4310	42.89
				λ=0.6851