

PLANNING FOR SUSTAINABLE DEVELOPMENT OF A RIVER BASIN

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ABSTRACT

The application of systems analysis techniques in evaluating project alternatives having more than one objective in river basin planning for the sustainable development is of recent origin. This paper presents the planning under fuzzy environment including both qualitative and quantitative aspects for the sustainable development of a major peninsular river basin (Krishna River Basin) in southern India. Formulation of alternatives include seven reservoirs and a diversion head work with eight main objectives. A general trapezoidal fuzzy weights and the fuzzy arithmetic were used for ranking the alternatives. Ranks of the alternatives and the formulation of Multi Objective Fuzzy Linear Programming (MOFLP) problem for the optimum utilization of the water resources for the preferred alternative ranked were also presented.

INTRODUCTION

In the last two decades there has been an increased awareness of the need to identify and consider simultaneously several objectives in the analysis and solution of problems, in particular those derived from large scale systems. Therefore, decision making in such multi disciplined, multi criterion (or multi objective) problems is one major problem facing the planners and Decision Maker (DM) as it involves decision making under conditions of risk and uncertainty. In most of the complex and complicated cases, the quantitative basis for decision making may be obtained by mathematical optimization methods (mathematical programming) while the final decision making should consider the whole series of various conditions and factors (i.e. qualitative aspects) such as legal, social, moral, prestige and those related to traditions, emotional aspects etc., which cannot be mathematically formulated. Therefore, the only possible method for solving such problems is the proper combination of the mathematical approach of OR and the intuitive knowledge and experience of the experts.

Traditionally, in Water Resources (WR) development, the design of projects is focused on the estimate of benefits and costs. A more realistic analysis would include environmental, social and regional objectives as well. The purpose of the Multi Criteria Decision Making (MCDM) methods are to help improve the quality of decision making more explicit, rational and efficient by providing information of the trade-off for the better understanding of the nature of choices. This helps in eliminating the alternatives dominated in every criterion by other alternatives. They also help the DM to articulate and apply their values to the problem rationally and consistently and to document the process that inspires the confidence in the soundness of the decisions. The political decision making process appropriate to many WR problems was described by Major (1969) and a valuable insight was offered by Haith and Loucks (1976). Haimes (1977) set forth the principles of regional WR planning to assist the policy decision making process at various hierarchical levels - local, state, regional and federal. A review and evaluation of MCDM methods was

presented by Cohon and Marks (1975).

The theoretical developments in MCDM methods (Zeleny, 1982; Roy, 1985; and Vincke, 1992), in fuzzy sets and systems (Zadeh, 1965; Dubois and Prade, 1980) and in artificial neural networks (Requena, 1992) paved the way for many application oriented papers to bridge the gap between theory and practice. Some of the notable applications of MCDM methods include Heidele and Duckstein (1983), Guariso et. al., (1986), Parent and Lebdi (1992), Tecele and Duckstein (1992) and Anand Raj and Nagesh Kumar (1996). Other notable contributions using fuzzy set theory were Jains' (1977) concept of maximizing set, membership level concept of Baas and Kwakernaak (1977), three indices proposed by Yager (1981), index of strict preference defined by Watson et. al., (1979), four grades of dominance studied by Adamos' (1980) and method of Kerre (1982). Cano et. al., (1991) and Requena et. al., (1994) had presented methods using artificial neural networks.

Most of these methods suffer at least from one of the following drawbacks: (1) The procedure is computationally complex and hence difficulty in implementation; (2) unintuitive, which also hinders implementation; (3) assume one criterion or one expert; (4) presupposes existence of some fuzzy relation or other function, across the alternatives or (5) produce a crisp ranking from fuzzy data. To overcome these difficulties and to make the problem simple and straight forward the authors (Anand Raj and Nagesh Kumar, 1996(a)) proposed a method (RANFUW: RANKing FUZZY Weights) and applied to a case study (Anand Raj and Nagesh Kumar, 1996(b)) for ranking the river basin planning alternatives. This method is intuitive, computationally simple and easy to implement. In this method the fuzzy weights of the alternatives were arrived at by the fuzzy information (opinions or preference structure) supplied by the multiple experts on the alternatives and the criteria. The alternatives were then ranked using the concepts of the Maximizing Set, the Minimizing Set and the Total Utility or Order Values. This methodology is briefly presented with the results of the case study. MOFLP problem formulation for the optimum utilization of WR with the preferred alternative and a numerical example are also presented at the end.

METHODOLOGY

The problem considered in this paper is to select the best alternative from amongst m alternatives (A_i ; $i = 1, 2, \dots, m$), with the help of the information supplied by n experts (E_j ; $j = 1, 2, \dots, n$) about the alternatives for each of K criteria (C_k ; $k = 1, 2, \dots, K$) and also the relative importance of each criterion with respect to an overall objective. There are four steps in this methodology.

Step (i): Defining and specifying the type of fuzzy numbers to be used by the experts

Let a_{ij}^k be the fuzzy number assigned to an alternative A_i by an expert E_j for the criterion C_k and c_{kj} be the fuzzy number given to criterion C_k by an expert E_j . Let these fuzzy numbers be a subset of \mathbf{F} described by

$$(1) \quad a_{ij}^k = (\alpha_{ij}^k / \beta_{ij}^k, \gamma_{ij}^k / \delta_{ij}^k) \quad \text{and} \quad c_{kj} = (\epsilon_{kj} / \zeta_{kj}, \eta_{kj} / \theta_{kj})$$

where $\alpha < \beta < \gamma < \delta$ and $\epsilon < \zeta < \eta < \theta \quad \in \quad L(0, 2, \dots, L)$

In equation (1) L is a linearly ordered continuous scale of preference information to be used by the

experts designed by the DM and L is a positive integer sufficiently large ($L = 10$ is used in this paper) to accommodate the information of the preference structure of the experts. Let $\mu_{A_i}(x)$ and $\mu_{C_k}(x)$ be the membership (general triangular) functions of a_{ij}^k and c_{kj} respectively.

Step (ii): Pooling, averaging or aggregating the fuzzy numbers across the experts

For pooling and averaging there are two ways. One way is to 'pool first'. In this method the DM averages the fuzzy numbers across the judges first and then weights are determined. The other way is to 'pool last', in which rankings of each alternative by each expert are computed first and then these ranks are pooled across the experts to determine weights. For pool first procedure we get

$$m_{ik} = 1/nO[a_{i1}^k \oplus a_{i2}^k \oplus \dots \oplus a_{in}^k] \text{ and } n_k = 1/nO[c_{k1} \oplus c_{k2} \oplus \dots \oplus c_{kn}] \quad (2)$$

In equation (2) \oplus represents the fuzzy addition and O represents the fuzzy multiplication.

Step (iii): Computing the fuzzy weights (w_i)

Given the above mentioned information the DM computes the fuzzy weights (w_i ; $i = 1, 2, \dots, m$) using

$$w_i = (1/KL)O[(m_{i1}On_1) \oplus (m_{i2}On_2) \oplus \dots \oplus (m_{ik}On_k)] \quad (3)$$

The fuzzy weight w_i could be computed using standard fuzzy arithmetic and then one could arrive at its membership function, $\mu_{w_i}(x) = (\alpha_i / \beta_i, \gamma_i / \delta_i)$. This function has a parabolic variation between (α_i, β_i) & (γ_i, δ_i) and a linear variation between (β_i, γ_i) .

Step (iv): Ranking of the alternatives

For ranking the alternatives, triangular membership functions of the maximizing set, $\{\mu_M(x)\}$ and the minimizing set, $\{\mu_m(x)\}$ were defined first. These functions could be linear, convex curved (risk-prone) or concave curved (risk-averse) and in general covers the three types of preferences: fair, adventurous and conservative, respectively. The total utility value or the order value, $\{U_T(i)\}$ of the membership function, $\mu_{w_i}(x)$ was then determined using

$$U_T(i) = \{U_M(i) + 1 - U_m(i)\} / 2 \quad (4)$$

where $U_M(i) = \sup_x \{\mu_{w_i}(x) \cap \mu_M(x)\}$ and $U_m(i) = \sup_x \{\mu_{w_i}(x) \cap \mu_m(x)\}$

Using $U_T(i)$ one can rank the alternatives. If two alternatives have the same utility values ($U_T(1) = U_T(2)$), one might use the vertices of the graphs of the corresponding membership functions to make the decision. (i.e., the vertex farther right is the largest, with decreasing size from the right to the left).

APPLICATION

The physical system considered in this study, the Krishna river basin, is one of the major peninsular river basins in India. The Krishna has a total length of 1400 km., and rises from a spring at Mahabaleswar and

flows through three states: Maharashtra, Karnataka and Andhra Pradesh. Its drainage area is of the order of 260 000 km². The important tributaries of this river are the Koyna, Ghataprabha, Malaprabha, Bhima and Tungabhadra. The river finally enters Bay of Bengal at Machilipatnam in Andhra Pradesh.

Most of the reservoirs in the basin are constructed as either single or multi purpose projects. The Bhadra, Tungabhadra, Nagarjuna Sagar and Ghataprabha are multi purpose (irrigation and hydropower are the major ones) projects while the Srisailem and Koyna are hydropower projects and the Almatti reservoir is an irrigation project. Increase in population densities and in the number of industries along the river course and around the reservoirs changed the land-use pattern over the years. This had resulted in the demand for water enormously. The resulting unplanned development in the basin to meet this demand led to the problems related to both quality and quantity of water such as waterlogging making a large portion of the irrigated area into unproductive, increase in alkalinity and salinity of subsoil resulting in health problems to both the human and the animal livestock which consumed the produce of the affected land, land submergence and associated rehabilitation problems etc. Therefore, a need arose for the development of the existing reservoirs and plan new reservoirs for the required water resource and to consider various conflicting objectives for the sustained development of the entire basin.

The purpose of the study was to find the most suitable planning of the reservoirs with their associated purposes aimed at the sustained development of the basin. For this purpose seven reservoirs and a diversion headwork were considered by Anand Raj and Nagesh Kumar (1996) for the formulation of 24 alternative systems (i.e., various combinations of reservoirs) with eight objectives which were further subdivided into 18 criteria. Of these 24 alternative systems, a subset of seven alternatives were found to be preferred over the others. These seven alternatives and the eight objectives are considered in this study. The Krishna river basin, the location of the reservoirs, their names and the preferred alternatives were shown in fig. 1. Table 1 gives the objectives and the sub criteria considered in the study.

RESULTS AND DISCUSSION

For the evaluation of the alternatives, three experts (E_j ; $j = 1,2,3$), one academician, one field engineer and an official from ministry of WR were consulted to give their opinion (preference structure) about the alternatives and the criteria in the form of fuzzy numbers between 0 and 10 ($L = 10$). Experts were supplied with the information about the reservoirs, alternatives and the associated purposes, advantages, disadvantages and other aspects. They were also supplied with the relevant information about the criteria for evaluation. A typical evaluation, for example, is given in table 2. It could be seen that the experts had given highest priority to the criterion C_1 and then to criterion C_7 , while least priority was given to the criteria C_6 and C_8 . Similar data was received about the alternatives for each of the criterion. Then the fuzzy weights (w_i ; $i = 1, 2, \dots, 7$) were determined using the eqn. (3) and were given by

$$\begin{aligned} w_1 &= (2.438 / 2.924, 3.121 / 3.708); & w_2 &= (1.911 / 2.419, 2.639 / 3.064) \\ w_3 &= (1.675 / 2.099, 2.265 / 2.699); & w_4 &= (2.826 / 3.367, 3.582 / 4.150) \\ w_5 &= (2.147 / 2.590, 2.715 / 3.168); & w_6 &= (1.778 / 2.257, 2.414 / 2.840) \\ w_7 &= (2.468 / 3.026, 3.156 / 3.650); \end{aligned}$$

The linear membership functions of the maximizing set and the minimizing set were given by

$$\mu_M(x) = \begin{cases} \{(x - 1.675) / (4.150 - 1.675)\} & 1.675 < x < 4.150 \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

$$\mu_m(x) = \begin{cases} \{(x - 4.150) / (1.675 - 4.150)\} & 1.675 < x < 4.150 \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

Membership functions of the fuzzy weight, $\{\mu_{wi}(x)\}$, the maximizing set, $\{\mu_M(x)\}$ and the minimizing set, $\{\mu_m(x)\}$ were shown in fig.2. Using eqn. (4) the total utility values, $(U_T(i))$ were determined and the alternatives were ranked as given in table 3. It was found that the alternative A_4 is the best and the alternative A_7 the next best while the alternative A_3 is the least preferred. Since a single continuous scale was used for all the alternatives and criteria, the final ranking was unique and changes in scale do not affect the results. More over the intuitive knowledge of the experts in addition to the quantitative data available was effectively used in the analysis. This kind of analysis is more rational and realistic.

MULTI OBJECTIVE FUZZY LINEAR PROGRAMMING (MOFLP)

It is seen that alternative A_4 ranked number one. The reservoirs in this alternative are not fully utilized to their potential. Therefore, we propose a MOFLP monthly (12 period) model for the optimal utilization of the WR available for this alternative. Schematic representation of this system is shown in fig. 3.

This system comprises of two reservoirs, one at Srisailam (R_3), the other at Nagarjuna Sagar (R_4) and a canal diversion work Prakasam Barrage, (R_5) at Vijayawada. Two power houses (nodes 1 & 2) are located on right and left banks of Srisailam reservoir and a regulator on the foreshore to supply water for irrigation in Rayalaseema and water supply to Madras (Telugu Ganga Project). Also for Nagarjuna Sagar, there are right and left power houses (nodes 3 & 4) in addition to a down stream power house (node 5).

River runoff is the inflow into Srisailam reservoir. Turbine releases, spills from this reservoir and the interflow into the river course between the two reservoirs comprise the inflows into Nagarjuna Sagar reservoir. Inflows into Prakasam Barrage comprises of turbine release into down stream turbine, spills from Nagarjuna Sagar reservoir and the interflow between R_4 and R_5 . Outflows from the reservoirs include releases for irrigation & turbines, spills and losses. Flows diverted through the power canals is also utilized to meet the down stream irrigation demands after flowing through the turbines at Nagarjuna Sagar.

Objective Function

The twin objectives considered are maximization of irrigation (RI) and hydropower production (RP).

$$Max. Z_1 = \sum_{j \in R_3, 3, R_4, R_5} \sum_{t=1}^{12} RI_{j,t} \quad (7)$$

$$and \quad Max. Z_2 = \sum_{j=1}^5 \sum_{t=1}^{12} RP_{j,t} \quad (8)$$

Where j refers to the node, t refers to the time period (month).

Constraints

(A) Turbine release - capacity constraints:

The release through the turbines for power production at nodes 1 to 5 in all periods ($t = 1, 2, \dots, 12$) should be less than or equal to the flow through capacity (T_{jc}) of the turbines at these nodes. Also power production in each month should be greater than or equal to the firm power (P_{jt}).

$$RP_{j,t} \leq T_{jc} \quad \text{for } j = 1, 2, \dots, 5 \text{ and } t = 1, 2, \dots, 12. \quad (9)$$

$$RP_{j,t} \geq P_{jt} \quad \text{for } j = 1, 2, \dots, 5 \text{ and } t = 1, 2, \dots, 12. \quad (10)$$

(B) Irrigation supply - demand constraints:

The diversion of canals to flow through turbines at node 4 and the reservoir releases R_3 , R_4 and R_5 should be greater than or equal to minimum irrigation demands (ID) at these points in all periods. That is

$$RI_{j,t} \geq ID_{j,t} \quad \text{for } j = R_3, 3, R_4, R_5 \text{ and } t = 1, 2, \dots, 12. \quad (11)$$

The release for irrigation at node R_4 (i.e., $RI_{R_4,t}$) is delivered through left canal power house at node 4 to generate power. The excess irrigation demand over the turbine capacity at node 4 is delivered through left canal, bypassing the turbine.

(C) Reservoir storage - capacity constraints:

The live storage in the reservoirs ($S_{j,t}$) at R_4 and R_5 should be less than or equal to the maximum live capacities (S_{jc}) respectively in all periods. That is

$$(12) \quad S_{j,t} \leq S_{jc} \quad \text{for } j = R_3 \text{ \& } R_4 \text{ and } t = 1,2,\dots,12.$$

(D) Reservoir storage continuity constraints:

These constraints relate the turbine releases (T), irrigation releases (RI), spills (O), reservoirs storage (S), inflows into the reservoirs (I) and losses (L) in all periods at R_3 , R_4 and R_5 . That is

$$S_{R3,t-1} + I_{R3,t} - RI_{R3,t} - RP_{1,t} - RP_{2,t} - O_{R3,t} - L_{R3,t} = S_{R3,t} \quad \text{for } t = 1,2,\dots,12. \quad (13)$$

$$S_{R4,t-1} + I_{R4,t} + RP_{1,t} + RP_{2,t} + O_{R3,t} - RI_{R4,t} - RP_{3,t} - RP_{4,t} - RP_{5,t} - O_{R4,t} - L_{R4,t} = S_{R4,t} \quad \text{for } t = 1,2,\dots,12. \quad (14)$$

$$I_{R5,t} + RP_{5,t} + O_{R4,t} - RI_{R5,t} - O_{R5,t} - L_{R5,t} = 0 \quad \text{for } t = 1,2,\dots,12. \quad (15)$$

Algorithm for MOFLP

To solve the model formulated, following algorithm is proposed. This algorithm has four steps. They are

Step 1: Solve the model as a Linear Programming (LP) problem by taking one objective at a time and find for each objective (Z_1 and Z_2) respectively, the best (Z_1^+ , Z_2^+) values and worst (Z_1^- , Z_2^-) values corresponding to the set (decision variables) of solutions (X^1 , X^2).

Step 2: Define a linear membership function $\mu_k(x)$ for each objective as

$$(17) \quad \mu_{Z_k}(x) = \begin{cases} 0 & Z_k \leq Z_k^- \\ (Z_k - Z_k^-) / (Z_k^+ - Z_k^-) & Z_k^- \leq Z_k \leq Z_k^+ \\ 1 & Z_k \geq Z_k^+ \end{cases} \quad \text{for } k = 1,2.$$

Step 3: An equivalent LP problem (crisp model) is then defined as

Maximize \ddot{e}

$$\text{subjected to } \ddot{e} \leq (Z_k - Z_k^-) / (Z_k^+ - Z_k^-) \quad \text{for } k = 1,2 \quad (18)$$

and all the original constraint set and non negativity constraints for X & \ddot{e}

Step 4: Solve the LP problem formulated in step 3. The solution is \ddot{e}^* (i.e., maximum degree of overall satisfaction) which is achieved for the solution X^* . The corresponding values of the objective functions Z_k^* are obtained and this is the optimal compromise solution.

NUMERICAL EXAMPLE

Objective Function: Maximize $Z_1 = -x_1 + 2x_2$ & $Z_2 = 2x_1 + x_2$
 subjected to: $-x_1 + 3x_2 \leq 21$; $x_1 + 3x_2 \leq 27$; $4x_1 + 3x_2 \leq 45$; $3x_1 + x_2 \leq 30$;
 and $x_1, x_2 \geq 0$ (where x_1, x_2 are the decision variables)

Step 1:

For the problem the solution is $X^1 = (0,7)^T$; $X^2 = (9,3)^T$ and $Z_1^- = -3$; $Z_1^+ = 14$; $Z_2^- = 7$ and $Z_2^+ = 21$.

Step 2:

$$\mu_{zk}(x) = \begin{cases} 0 & Z_1(x) \leq -3; \\ (Z_1(x) + 3) / 17 & -3 \leq Z_1(x) \leq 14; \\ 1 & 14 \geq Z_1(x); \end{cases} \quad \mu_{zk}(x) = \begin{cases} 0 & Z_2(x) \leq 7; \\ (Z_2(x) - 7) / 14 & 7 \leq Z_2(x) \leq 21; \\ 1 & 21 \geq Z_2(x); \end{cases}$$

Step 3: Now formulate the crisp model for the example as

Maximize \ddot{e}
 subjected to: $\ddot{e} \leq -0.05882x + 0.11764x + 0.17646$
 $\ddot{e} \leq 0.14286x + 0.07142x + 0.5$
 $-x_1 + 3x_2 \leq 21$; $x_1 + 3x_2 \leq 27$; $4x_1 + 3x_2 \leq 45$; $3x_1 + x_2 \leq 30$;
 and x_1, x_2 and $\ddot{e} \geq 0$.

Step 4: The solution for the problem is

$\ddot{e}^* = 0.74$ (overall satisfaction); $X^* = (5.03, 7.32)^T$ and $Z_1^* = 17.38$ and $Z_2^* = 4.58$.

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