OPTIMAL RESERVOIR OPERATION USING FUZZY APPROACH

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ABSTRACT

Optimal reservoir operation model is developed using Multi Objective Fuzzy Linear Programming (MOFLP) which is computationally simple and easy to implement to the real world situation of reservoir operation. Application of MOFLP is demonstrated through the case study, Hirakud reservoir in Orissa State, India. Problem is formulated with two objective functions viz. maximization of releases for irrigation and maximization of hydro power produced, with several constraints and is solved in an iterative manner. Linear membership functions are used to fuzzify the objective functions. Only objectives are taken to be fuzzy and all other parameters of the model are considered crisp in nature. MOFLP is used to obtain a compromise solution simultaneously optimizing the fuzzified objectives and the level of satisfaction. For different satisfaction levels of both the objectives, MOFLP model is run to provide alternative scenarios to the decision maker. Optimal policies were also determined for various inflow scenarios using MOFLP. These optimal policies can be implemented by the reservoir authorities based on the qualitative prediction of inflows into the reservoir.

INTRODUCTION

Since the advent of operations research as a scientific approach to optimization, a variety of mathematical tools have been developed and applied to problems in diverse fields of science and technology including water resources. Several approaches for optimal reservoir operation were developed and some of their limitations were well discussed in the literature (e.g.: Yeh, 1985).

There are basically three approaches to the solution of problems with multiple objectives: (1) Find the preferred solution directly or (2) First generate a non-inferior set and then find the preferred solution from among them or (3) System analyst could be concerned only with the development of non-inferior solutions and then the decision maker could choose, according to his values and requirements, one or more of these solutions for implementation. Many approaches were developed for multi objective optimization such as deterministic approach, stochastic approach (to deal with randomness), fuzzy-based approach (to deal with vagueness and imprecision), expert systems (to incorporate expert's intuitive knowledge and expertise), artificial neural network based models (data driven models, in contrast to the model driven type), among others.

Multi Objective Analysis in water resources has developed in explicit form largely through the work of Harvard Water Program (HWP). Much of the methodology and its research findings were published by Mass et al., (1962). Non-fuzzy multi objective approaches include Vedula and Rogers (1981) and Srinivasa Raju and Nagesh Kumar (1999).

To overcome some of the limitations in previous approaches, fuzzy based models were proposed. Shrestha et al., (1996) introduced a fuzzy-rule based model, deriving the operation rules for a multi-purpose reservoir. Russell and Campbell (1996) proposed operating rules for a single purpose hydroelectric project, where both the inflows and selling prices of energy are uncertain. Anand Raj and Nagesh Kumar (1998 and 1999) proposed fuzzy based approach, RANFUW, for ranking multi criterion river basin planning alternatives using fuzzy numbers and weights.

In the present study, Multi Objective Fuzzy Liner Programming (MOFLP) is used based on Zimmerman's (1978) vector maximization approach. Although Zimmerman's approach considers both objectives and constraints as fuzzy, in the present study, only objectives are considered as fuzzy. As can be seen from the MOFLP formulation in the following sections, annual releases for irrigation and power are considered fuzzy (through objective functions) while monthly releases were considered crisp. This approach gives only preliminary results and for detailed investigation other decision variables should also be considered as fuzzy variables.

ALGORITHM FOR MOFLP

To solve the MOFLP model, the following algorithm (for maximisation problem) can be used (similar algorithm can be easily developed for minimisation problem).

Step 1: Solve the model as a Linear Programming (LP) problem by taking one objective at a time and find for each objective (Z_k) respectively, the best (Z_k^{+}) values and worst (Z_k^{-}) values corresponding to the set (decision variables) of solutions (X_k*).

Step 2: Define a linear membership function μ_k (x) for each objective as

$$\mu_{zk} = \begin{cases} 0 & Z_k \le Z_k^- \\ (Z_k - Z_k^-) / (Z_k^+ - Z_k^-) & Z_k^- \le Z_k \le Z_k^+ \quad \text{for } k = 1,2... \\ 1 & Z_k \ge Z_k^+ \end{cases} \qquad (1)$$

Step 3: An equivalent LP problem (crisp model) is then defined as

Maximize ς

subject to $\varsigma \le (Z_k - Z_k^-) / (Z_k^+ - Z_k^-)$ for k= 1,2 … (2)

and all the original constraint sets and non negativity constraints for X & ς.

Step 4: Solve the LP problem formulated in step 3. The solution is ς^* (i.e., maximum degree of overall satisfaction) which is achieved for the solution X^*. The corresponding values of the objective functions Z_k^* are obtained and this is the best compromise solution.

The methodology for fuzzy optimization as explained is applied to the case study, to develop the optimal operating policy.

CASE STUDY

The physical system considered is Hirakud reservoir, a multi purpose project, created by constructing a dam across the river Mahanadi in Sambalpur district, Orissa state, India. Hirakud dam is situated at latitude of 21^0 32' N and longitude 83^0 52' E. It is mainly constructed for flood control with additional features of irrigation and hydro power generation. The reservoir has a catchment area of 83,400 sq.km. and a storage capacity of 7190.856 M.cu.m. The reservoir provides irrigation to 1554.01 sq.km. in Kharif season and to 1082.09 sq.km. in Rabi season totaling to annual irrigation of 2635.89 sq.km. For power generation the total installed capacity is 307.5 MW.

Average monthly inflows into the reservoir are shown in Table 1. Monthly irrigation demands were determined with the help of crop calendar, water requirements for different crops during different growth stages and the types of soils. Average net irrigation demands for each month are also shown in the Table 1.

Table 1. Monthly inflows and irrigation demands for Hirakud reservoir

Month	Inflows in Mcum	Irrigation demand in Mcum
January	188.649	200.979
February	134.397	212.076
March	94.941	256.464
April	48.087	242.901
May	24.660	45.621
June	1203.408	61.650
July	7462.116	177.552
August	12893.480	196.047
September	8353.575	225.639
October	2329.137	257.697
November	589.374	87.543
December	244.134	113.436

FORMULATION OF MOFLP MODEL

The MOFLP model is developed for monthly operation of the reservoir assuming stationary inflows and average monthly demands. Here the objective functions are considered as fuzzy and the constraints are considered as non-fuzzy.

Objective functions

The two objectives considered in the study are: (1) Maximization of releases for irrigation (i.e., RI), and (2) Maximization of hydro power production (i.e., PP).

$$\text{Max } Z_1 = \text{Max (TOTRI)} \quad (3)$$

$$\text{Max } Z_2 = \text{Max (TOTPP)} \quad (4)$$

where TOTRI is the total releases for irrigation in all the time periods (i.e., months), and TOTPP is the total hydro power produced in all the time periods. These objective functions can be written as

$$Max\ Z_1 = \sum_{t=1}^{12} RI_t \qquad (5)$$

$$Max\ Z_2 = \sum_{t=1}^{12} PP_t \qquad (6)$$

Since a linear programming algorithm is used, the non-linear relationships involving the product of releases and head to compute power produced should be replaced by a linear relationship. If the average head h_t^o and average release q_t^o can be estimated for each period t, then these fixed constants can be used to obtain a linear relationship of the release-head product term following Loucks et al., (1981).

$$q_t h_t = q_t^o h_t^o + q_t^o (h_t - h_t^o) + h_t^o (q_t - q_t^o)$$

$$q_t h_t = q_t^o h_t + h_t^o q_t - q_t^o h_t^o \qquad (7)$$

For a 30-day month, power produced in a month in Mega Watt Hours (MWH) is given by

$$PP_t = 2.725 \{q_t^o h_t + h_t^o q_t - q_t^o h_t^o\} \qquad (8)$$

Constraints

Turbine Release Constraint

The Hydro power production (PP) by the turbine in each month (t), should be less than or equal to the turbine capacity (TCAP), and also it should be greater than or equal to the firm power (FP) committed for that month.

$$PP_t \le TCAP \qquad \forall\ t = 1,2\ldots\ldots,12 \qquad (9)$$

$$PP_t \ge FP_t \qquad \forall\ t = 1,2\ldots\ldots,12 \qquad (10)$$

Irrigation demand constraint

Release into canals for irrigation (RI) should be less than or equal to irrigation demand (ID). Release should also be greater than minimum irrigation required so that the crop will not wilt (in the present case, 20% of the irrigation demand is considered as minimum irrigation demand) for all the time periods.

$$RI_t \le ID_t \qquad \forall\ t = 1,2\ldots\ldots,12 \qquad (11)$$

$$RI_t \ge 0.2\ ID_t \qquad \forall\ t = 1,2\ldots\ldots,12 \qquad (12)$$

Reservoir storage capacity constraint

The live storage in the reservoir should be less than or equal to the maximum capacity (SCAP) for all the time periods.

$$S_t \le SCAP \qquad \forall\ t = 1,2\ldots\ldots\ldots 12. \qquad (13)$$

Reservoir storage continuity constraint

These constraints relate to the releases for the turbine (q), releases for irrigation (RI), reservoir storage (S), inflows (I) into the reservoir, overflows (O) and the evaporation losses (L) for all the time periods.

$$S_t + I_t - RI_t - q_t - O_t - L_t = S_{t+1} \qquad (14)$$

By considering the evaporation losses as a function of storage (Loucks et al., 1981) and by assuming a linear relationship between reservoir water surface area and storage, storage continuity constraint can be written as follows.

$$(1-a_t)\ S_t + I_t - RI_t - q_t - O_t - A_o\ e_t = (1+ a_t)\ S_{t+1} \qquad (15)$$

where

$a_t = A_a\ e_t / 2$

A_a is surface area of the reservoir per unit active storage volume

A_o is surface area of the reservoir corresponding to the dead storage volume

e_t is evaporation rate for month t in depth units

In this constraint, when last month of the year (i.e., t = 12) is considered, (t = 13) refers to the first month of the next year.

The multi objective linear programming model formulated in this section is applied to the case study, and is solved using LINGO (Language for INteractive General Optimization), in an iterative manner, for a compromise solution and for various satisfaction levels (i.e., U). Thus a set of alternative solutions are obtained for the decision maker to take a decision to suit his requirements.

RESULTS AND DISCUSSIONS

By adopting the MOFLP algorithm already explained, the best and the worst values (Z^+ and Z^-) for both the objectives (viz. Z_1: Releases for irrigation and Z_2: Hydro power produced) are determined by considering one objective at a time. A program implemented in LINGO is used for this purpose. When Z_1 is maximized, the corresponding value of Z_2 is considered to be the worst and vice-versa. These values are given in Table 2.

Table 2. Best and worst values of the objective function

Objective function	Best/ worst values	
	Best value (Z^+)	Worst value (Z^-)
Release for Irrigation (RI) in Mcum	2077.605	856.935
Hydro Power Produced (PP) in GWH	1262.746	1108.987

Once the upper and lower limits of the objective functions are determined, in the second step, objective functions are fuzzified by considering suitable membership function. In the present study, linear membership functions are considered. The membership functions for both the objectives Z_1 and Z_2 are shown in Figures 1 and 2 respectively and can be stated as follows.

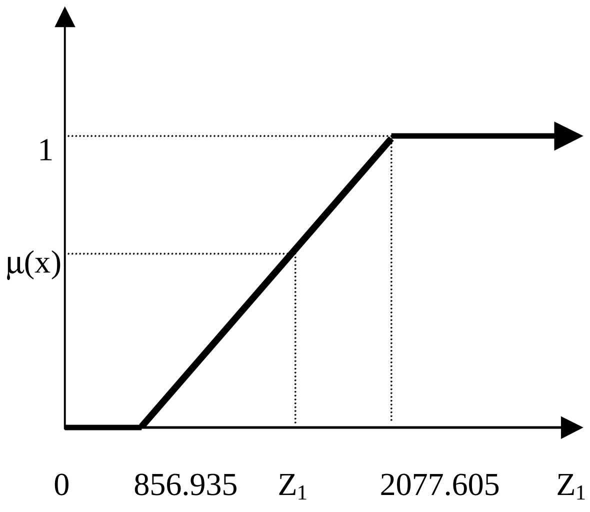

Figure 1. Membership function for Z_1

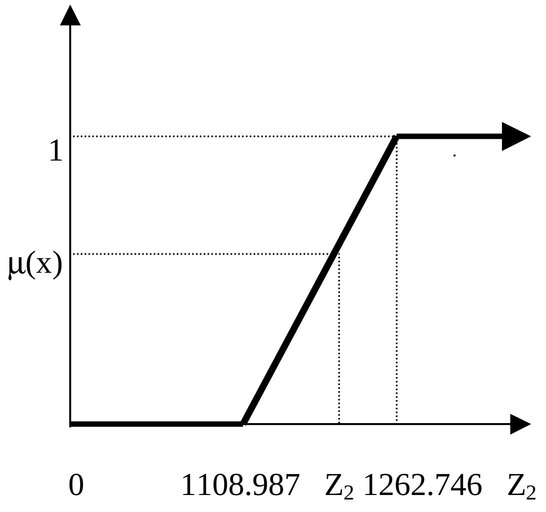

Figure 2. Membership function for Z_2

$$\mu_{Z_1}(x) = \begin{cases} 0 & Z_1 \leq 856.935 \\ \left(\dfrac{Z_1 - 856.935}{2077.605 - 856.935} \right) & 856.935 \leq Z_1 \leq 2077.605 \\ 1 & Z_1 \geq 2077.605 \end{cases} \qquad (16)$$

$$\mu_{Z_2}(x) = \begin{cases} 0 & Z_2 \leq 1108.987 \\ \left(\dfrac{Z_2 - 1108.987}{1262.746 - 1108.9871} \right) & 1108.987 \leq Z_2 \leq 1262.746 \\ 1 & Z_2 \geq 1262.746 \end{cases} \quad (17)$$

By incorporating the above information, the following modified LP problem is formulated as the third step of the algorithm. Coefficients for constrains given below are obtained from equations 16 and 17.

Max U

subject to $U \leq 0.0008192\ Z_1 - 0.702$

$U \leq 0.000006504\ Z_2 - 7.212$

and all the original constraints given in the model and $U \geq 0$

In this formulation, U is the level of satisfaction derived by simultaneously optimizing the fuzzified objectives Z_1 and Z_2. The solution of this LP model is found in the next step. The results obtained are as follows.

U (maximum level of satisfaction) = 0.594

$Z_1^* = 1582.323$ and $Z_2^* = 1200.221$

The optimal operation policy (i.e., monthly releases and hydro power produced) thus obtained is given in Table 3.

Table 3. Optimal operating policy for maximized satisfaction level

Month	Hydro Power (PP) Produced in GWH	Release for Irrigation (RI) in Mcum	Total PP in GWH	Total RI in Mcum
January	169.560	40.195		
February	27.000	42.415		
March	27.000	252.408		
April	27.000	242.901		
May	27.000	45.621		
June	68.828	61.650	1200.221	1582.323
July	169.560	177.552		
August	169.560	196.047		
September	169.560	225.639		
October	169.560	257.697		
November	43.330	17.508		
December	13.226	22.687		

If the decision maker is satisfied with the values of U and the corresponding results, he can straight away adopt the results. Otherwise, he can change the satisfaction levels for both the objectives suitably and run the model again to get the solutions. The solution is to be checked and if found not satisfactory the satisfaction levels are changed in an iterative manner and the best suitable policy is arrived at. For this purpose a whole range of satisfaction levels for Z_1 (i.e., U_1 = 0.0 to 1.0) are considered and the corresponding U_2 values for Z_2 and the corresponding solutions are determined. Objective function values for different satisfaction levels are given in Table 4.

Table 4. Objective functions values for different satisfaction levels

Sl. No.	Degree of Satisfaction (U)		Objective Value (Z)	
	U_1	U_2	Z_1 in Mcum	Z_2 in GWH
1.	0.0	0.862	856.935	1262.746
2.	0.3	0.798	1233.144	1231.561
3.	0.5	0.662	1467.285	1210.365
4.	0.594	0.594	1582.323	1200.221
5.	0.7	0.518	1711.426	1188.557
6.	0.8	0.445	1833.496	1177.348
7.	0.9	0.372	1955.566	1166.014
8.	1.0	0.297	2077.605	1154.481

MOFLP model is run for different inflow patterns, which are 20% less than, 40% less than, 20% more than and 40% more than the annual average inflows. For this purpose, representative years from historic data are identified whose annual flows are 20% less than, 40% less than, 20% more than and 40% more than the annual average inflows and the corresponding monthly flows are used. A better dissaggregation approach (Nagesh Kumar et. al., 2000) can be used for more representative inflow scenarios. The optimal operation policies with these inflows are found out and the maximized satisfaction level and the corresponding maximized values of releases for irrigation and hydro-power produced are presented in Table 5.

Table 5. Maximized values of the objective functions for different inflow scenarios

Inflows for Year	Satisfaction Level (U)	Releases for Irrigation in Mcum	Hydro Power Produced in GWH
40% below average	0.701	1635.215	1040.018
20% below average	0.601	1500.800	1140.397
Average inflows	0.594	1582.323	1200.221
20% above average	0.544	1521.290	1247.577
40% above average	0.754	1776.990	1266.530

CONCLUSIONS

Multi Objective Fuzzy Linear Program (MOFLP) model is formulated for the optimal reservoir operation with multiple objectives. MOFLP model is applied to the case study, Hirakud reservoir on Mahanadi River in Orissa State, India, to obtain the optimal monthly operation policy. The objective functions considered are maximization of releases for irrigation and hydro power produced. Optimal operating policy obtained using MOFLP for the maximum level of satisfaction (U) was given in detail. A whole range of reservoir operation policies, for different satisfaction levels were also determined. Optimal policies were also determined for various inflow scenarios using MOFLP. These optimal policies can be implemented for better utilization of the water resources depending on the priority for each objective chosen by the decision maker.

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