STOCHASTIC LINEAR PROGRAMMING FOR OPTIMAL RESERVOIR OPERATION: A CASE STUDY

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ABSTRACT: Optimal reservoir operating policy should consider the uncertainty associated with the uncontrolled inflow to the reservoir. In the present study, Stochastic Linear Programming (SLP) model is developed to obtain optimal operating policy for the existing multipurpose reservoir. The objective of the model is to maximise the expected value of the system performance which is the sum of the all performance value times the respective joint probabilities. The decision variables are the probabilities of reservoir release, $PR_{k,i,l,t}$, which are the joint probabilities of the reservoir release, $R_{k,i,l,t}$, with an initial reservoir storage volume of $S_{k,t}$, inflow of $Q_{i,t}$ and the final storage volume of $S_{l,t+1}$ for a given time period t. The joint probabilities are influenced by the target reservoir storages, releases and the stochastic nature of inflows. This objective is subject to a set of stochastic constraints to maintain continuity. Historic inflow data is used to consider the stochastic nature of inflows in the form of inflow transition probability matrices. A computer program is developed in LINGO (Language for INteractive General Optimisation) to perform stochastic optimisation. The model gives the steady state probabilities of reservoir storage and inflow as output. The model is applied to Hirakud reservoir in Mahanadi river basin of orissa state, India, for development of an optimal reservoir operating policy. The steady state optimal operating policy and its implications are discussed in this paper.

Key Words: Stochastic optimisation, Stochastic linear programming, Steady state probabilities, Reservoir operating policy.

1. INTRODUCTION

The problem of determination of best allocation and utilisation of available limited resources is as old as man himself. The uncertainty of resources about the future water availability adds more complexity to the problem of optimum allocation. This allocation problem is being studied by economists, engineers and mathematicians for over last four decades under the perview of stochastic optimisation, which can be basically classified into Implicit Stochastic optimisation (ISO) and Explicit Stochastic Optimisation (ESO). A comprehensive state-of-the-art review of the mathematical models developed for reservoir operation was made by Yeh [8]. The approach for inclusion of uncertainty in Linear Programming (LP) formulation was first given by Dantzig. Manne [4] has demonstrated a sequential probability model by ESO in terms of an initial decision rule and Markov process, which was applied to inventory control problem. He tried to obtain the optimum expected value of objective function along with statistical properties of stochastic inputs to arrive at optimum decision. In water resources management, Watermeyer and Thomas [7] are first to incorporate the uncertainty in optimum decision. The ESO in form of Stochastic Linear Programming (SLP) and Stochastic Dynamic programming (SDP) were applied for evaluating sequential policy for multipurpose reservoirs during late sixties. SLP was applied for five Finger lakes of New York state by Loucks [2], for lake Seneca of New York state by Gablinger and Loucks [1]. LP has the advantage over DP with the availability of standard well defined algorithm and also there is no dimensionality problem [6]. Even then, SLP was not so attractive to use for a long time because a separate program was needed to incorporate the uncertainty of inflow and to insert the LP parameters in the LP package [1, 2]. With the availability of powerful packages to solve SLP problems, such as LINGO with derived sets, sparse sets features, the above difficulties were overcome, and the application of SLP to practical problems became easy to handle, which is demonstrated in this paper.

The Hirakud multipurpose reservoir, under this case study, is operated considering only flood control as the major objective. There is no prescribed operating policy for other purposes. So operating policy is available only for monsoon season but not for the non-monsoon season. An attempt is made to use the stochastic optimization model, SLP, which takes care of the uncertainty of uncontrolled inflow to the reservoir to obtain the operation policy on monthly basis throughout the year. The next section deals with the basic structure of SLP. Third section deals with the application of model to Hirakud multipurpose project. Fourth section shows the result of the optimization model and the derived operating policy.

2. STOCHASTIC LINEAR PROGRAMMING MODEL

Each and every component of a deterministic LP is considered as deterministic in nature. But in real practice, this never happens. Consider the transition of reservoir storage from one volume in one season to some other volume in next season. The transition results partly from the release for various uses, which can be controlled, and partly from inflow to the reservoir and reservoir losses, such as evaporation and seepage loss, which can not be controlled. So the first component can be made deterministic, but not the last two. They are random by its nature. Inclusion of such random components makes the LP formulation SLP. Reservoir storage volume, inflow, release and time are continuous variables, which are assumed to be discrete to simplify the computation (first order Markov process). The following notations are used in this SLP model, which should be explained before formulation.

In this model, Q_t represents random unregulated inflow into the reservoir. S_t and R_t represent random initial reservoir storage and release respectively, where t is the time period. The initial reservoir storage volume and possible inflow in each period are discretised and denoted by k and i indices. $PQ_{i,t}$ is the probability that the inflow in period t is within the discrete value range of the state i represented by inflow Q_i. Similarly, $PS_{k,t}$ is the probability that the initial reservoir storage volume is within state k represented by S_k discrete value. The state of inflow in the next time period i.e., t+1 is represented by j. The state of reservoir storage volume at the end of time period t or reservoir storage volume at beginning of time period t+1 is represented by l. The state of reservoir storage volume at the end of time period t+1 is represented by m. Loss from the reservoir in a period, which is considered as function of initial and final reservoir storage volumes, is represented by $E_{k,l,t}$. Release in a period t, which is dependent on initial reservoir storage volume, final reservoir storage volume and inflow, is given by

$$R_{k,i,l,t} = S_{k,t} + Q_{i,t} - E_{k,l,t} - S_{l,t+1} \quad (1)$$

Let $PR_{k,i,l,t}$ represent the joint probability of release for the given initial storage volume $S_{k,t}$, inflow $Q_{i,t}$ and final reservoir storage volume $S_{l,t+1}$. Let $B_{k,i,l,t}$ be the sum of the squared deviations from target storage volume and target release given by the equation (8). The objective is to minimise the total expected value of $B_{k,i,l,t}$. The SLP formulation is as given below.

Minimise $$\sum_k \sum_i \sum_l \sum_t (B_{k,i,l,t} \; PR_{k,i,l,t}) \quad (2)$$

subject to $$\sum_m PR_{l,j,m,t+1} = \sum_k \sum_i (PR_{k,i,l,t} \;\; P^t_{i,j}) \qquad \forall \; l, j, t \quad (3)$$

$$\sum_k \sum_i \sum_l PR_{k,i,l,t} = 1 \qquad \forall \; t \quad (4)$$

.

$$PR_{k,i,l,t} \geq 0 \qquad \forall \; k, i, l, t \quad (5)$$

(Non - negative constraints)

where $P^t_{i,j}$ is the transition probability of inflow to state j in month t+1 given that the state is i in the month t. In the above formulation, the objective is to minimise the total sum of the products of $B_{k,i,l,t}$ and its corresponding joint probability, which is nothing but the expected value of system performance. The left hand term of equation (3) defines the joint probability of storage volume state, l at the beginning of the period t+1, $S_{l,t+1}$, and inflow $Q_{j,t+1}$.The right hand term defines the joint probability of storage volume state, l at the end of the period t, $S_{l,t+1,}$ and inflow $Q_{j,t+1}$ .These two terms should be equal for continuity. When t = t_{max}, then t+1 = 1; i.e., the first period of next year is followed by the last period of current year. Constraint (4) states that all joint probabilities add up to 1 for each time period.

The steady - state probabilities of storage volume, $PS_{k,t}$, can be found out from $PR_{k,i,l,t}$ as follow.

$$PS_{k,t} = \sum_i \sum_l PR_{k,i,l,t} \qquad \forall \; k, t \quad (6)$$

3. MODEL APPLICATION

The stochastic linear programming model formulated in section 2 is applied to an existing reservoir namely, Hirakud reservoir [5]. The Hirakud multipurpose project is built across river Mahanadi at latitude $21^0$32' N, longitude $63^0$52' E in the Orissa state, India. Mahanadi river collects runoff from a catchment area of about 1,41,600 sq. kms. and joins the Bay of Bengal. In the process, it creates devastating floods in the costal districts, while four districts in the upstream side were often facing drought due to erratic rainfall pattern. Considering these points, a dam was built at Hirakud in 1956, incorporating flood control, irrigation and hydropower as various purposes in that order of preference.

Besides flood protection to 9,500 sq. kms. of delta area, 1,55,635 ha of area in Kharif season, 1,08,385 ha of area in Rabi season is irrigated and 307.5 MW of installed capacity of hydropower is generated from this multipurpose project. The average annual rainfall in the basin is about 1420 mm. The daily minimum temperature in winter season varies from 7^0 C to 12.8^0 C, where as the daily maximum temperature in summer season varies from 42.9^0 C to 45.5^0 C.

Thirty five years of monthly data is used in this study. The data used are inflow to the reservoir, initial storage volume, evaporation and release for irrigation and power. Monthly total volume of all the above data are expressed in Million Acre-feet (M. ac. ft) units. The data is processed for the model and the optimal operating policy is obtained as explained in the following steps.

Step 1: The inflow volume of each month is discretised into 3 unequal states. The representative value for each state, $Q_{i,t}$, is found. The transition probability matrix for each month, $P^t_{i,j}$, is computed.

Step 2: The initial reservoir storage volume for each month is also discretised, and the representative value for each state, $S_{k,t}$, is found out. The reservoir should be full during later part of monsoon season i.e., in the months September and October. In the last forty years of operation, it was also observed that the reservoir is almost full during these two months. So three states in each month is chosen except for September and October months for which there is only one state (the reservoir capacity) for discretisation.

Step 3: A quadratic relationship between the water spread area (A) and active storage volume (S) data was established and is as follows.

$$A = 0.056 + 0.033 * S - 0.003 * S^2 \qquad (7)$$

Knowing the evaporation rate in the reservoir, total evaporation loss is found out for corresponding storage with the help of equation (7).

Step 4: $R_{k,i,l,t}$ value is found out for all possible combinations of states of initial reservoir storage volume, inflow to reservoir and final reservoir storage volume using equation (1), considering the evaporation losses, which is taken as function of average of initial and final reservoir storage volume.

Target initial storage volume (T^t_s) and target release (T^t_r) for each month t should be set depending upon the requirement of command area and other uses. But in this work, average of past thirty five years of release data is considered as target release and is shown in the Table 1. The target storage for each month is set judiciously considering the flood control in monsoon season and is shown with its corresponding state in Table 2. As the flood control is the most preferred purpose of this project, the reservoir should be empty (i.e., in state 1) in the month of July. The reservoir should be filled up to its capacity in the month of November (i. e. in state 3) to make the water available for the following non-monsoon seasons. In the remaining non-monsoon periods depending on target release, the target storage is set. For various combinations of states, $B_{k,i,l,t}$, summation of squared deviations from T^t_s and T^t_r is found out by equation (8).

$$B_{k,i,l,t} = [(R_{k,i,l,t} - T^t_r)^2 + (S_{k,t} - T^t_s)^2] \qquad (8)$$

If $R_{k,i,l,t}$ is more than the upper limit of release (the canal carrying capacity), then $R_{k,i,l,t}$ is restricted to the upper limit value. If $R_{k,i,l,t}$ is negative for a particular combination, the corresponding $B_{k,i,l,t}$ value is forced to a very high value, so that, that particular combination will not be considered in the optimal operating policy.

Table 1. Target release for every month.

Month	Target release (M. ac. ft)		Month	Target release (M. ac. ft)		Month	Target release (M. ac. ft)
JAN	0.62		MAY	0.50		SEP	1.04
FEB	0.59		JUN	0.51		OCT	0.88
MAR	0.70		JUL	0.94		NOV	0.56
APR	0.69		AUG	1.12		DEC	0.55

Table 2. Target storage for each month.

Month	Initial Storage Volume (M. ac. ft)	State		Month	Initial Storage Volume (M. ac. ft)	State
JAN	3.14	2		JUL	0.03	1
FEB	2.60	1		AUG	3.51	2
MAR	2.04	1		SEP	4.35	1
APR	1.33	1		OCT	4.35	1
MAY	0.60	1		NOV	4.36	3
JUN	0.58	2		DEC	4.09	2

Step 5: Equation (2) through (5) is solved by LINGO. In this particular formulation, there are 284 decision variables and 92 constraint equations. By the available derived sets and sparse sets and some other new features, the above formulation could be implemented easily. No separate program is necessary to handle uncertainty and LP parameters as before. The input to the program are $B_{k,i,l,t}$ and $P^{t}_{i,j}$. The decision variable values of $PR_{k,i,l,t}$ are obtained as output, from which the operating policy is found out.

Step 6: The operating policy is in the form of final reservoir storage volume i.e., $l^*=l\,(k,i,t)$. The optimal final reservoir storage volume is shown as the function of initial storage volume, inflow and time period t. The optimal release is then obtained by the equation (1). The operating policy function l (k,i,t) is obtained from the non-zero $PR_{k,i,l,t}$ values.

4. RESULTS AND DISCUSSIONS

The optimal operating policy function $l^* = l(k,i,t)$, obtained by solving SLP model, is given in Table 3. This policy is the steady state operating policy. The target storage and target release can not be achieved for each and every month. The optimum achievements are as shown in table 3 in terms of reservoir storage volume, and hence the release by equation (1), which satisfies the objective of minimisation of expected system performance represented by equation (2). The target storage is fully met in the months from February to May, July, September and October; is partly met in August, November and December; and is never met in January and June. If the reservoir is operated violating the optimal operating policy to satisfy the target, there may be short term gain, but in the long term, this option will turn out to be suboptimal.

Table 3. Optimal operating policy for each month.

JANUARY

	Inflow		
Storage	1	2	3
1	1	1	1
2	-	-	-
3	-	-	-

FEBRUARY

	Inflow		
Storage	1	2	3
1	1	1	1
2	-	-	-
3	-	-	-

MARCH

	Inflow		
Storage	1	2	3
1	1	1	1
2	-	-	-
3	-	-	-

APRIL

	Inflow		
Storage	1	2	3
1	1	1	1
2	-	-	-
3	-	-	-

MAY

	Inflow		
Storage	1	2	3
1	1	1	1
2	-	-	-
3	-	-	-

JUNE

	Inflow		
Storage	1	2	3
1	1	1	1
2	-	-	-
3	-	-	-

JULY

	Inflow		
Storage	1	2	3
1	1	2	2
2	-	-	-
3	-	-	-

AUGUST

	Inflow		
Storage	1	2	3
1	1	1	-
2	1	1	1
3	-	-	-

SEPTEMBER

	Inflow		
Storage	1	2	3
1	1	1	1

OCTOBER

	Inflow		
Storage	1	2	3
1	2	3	3

NOVEMBER

	Inflow		
Storage	1	2	3
1	-	-	-
2	2	2	3
3	2	2	-

DECEMBER

	Inflow		
Storage	1	2	3
1	-	-	-
2	1	1	1
3	-	1	1

The optimal operating policy for each month usually gives the policy for only those initial storage volume state, to which the reservoir reaches in the previous month. For example, the final storage volume state for the month of July can be 1 or 2 depending on the inflow state in that month (see table 3). So the policy for the month of August shows for the initial storage volume states 1 and 2 only. Policy is not defined for the other states (denoted by "-"), which are transient states. After steady state is reached, the reservoir storage will not reach the trasient state as it will result in suboptimal solution. Therefore, the remaining operation will involve only those states mentioned in steady state operating policy after steady state is reached.

5. SUMMARY

The application of Stochastic Linear Programming (SLP) for finding a suitable operating policy for Hirakud reservoir in Mahanadi river basin of Orissa state in India is presented in this paper. The operating policy is on monthly basis. From the available monthly data for thirty five years, the probability transition matrix of unregulated inflow to the reservoir is found out. Depending on the actual command area requirement, target reservoir storage and target release for every month is set. System (Project) performance is evaluated for all possible operations as the sum of squared deviations from target storage and target release. Then an optimal operating policy is found out by running the optimization model, SLP, on LINGO, which gives the optimum expected system performance. This monthly operating policy can be used on real time basis to get maximum long term benefits from the reservoir system.

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