

Inverse reliability based structural design for system dependent critical earthquake loads

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Abstract

The problem of reliability based design of structures subjected to partially specified random earthquake loads is considered. A procedure for the determination of structure–excitation pair that maximizes a specified response variable and, at the same time, achieves a target reliability is outlined. The procedure combines concepts from inverse first order reliability methods and methods for determining random critical earthquake loads. The formulation is shown to lead to a problem in constrained non-linear optimization. Issues related to spatial variability of earthquake loads are also addressed. Illustrative examples on a singly supported multi-degree of freedom system and a doubly supported single degree of freedom system are included.

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1. Introduction

The problem of specification of design earthquake loads for important structures is one of the most crucial and yet difficult problems in earthquake engineering. Here there are three needs that are to be taken into account: (a) the need to design important structures, such as nuclear power plants and large dams, to ensure a high level of reliability, (b) the need to have satisfactory models for the seismic inputs that are complete enough to carry out a desired response/reliability analysis, and (c) the need to have an adequate database of recorded earthquake ground motions to identify parameters for the input model. Often, these issues are inter-related in a complex manner and this calls for compromises in analysis and design. Thus, in the absence of adequate database on recorded strong motions, one would not be able to satisfactorily specify parameters of the input model and, this, in turn, makes the task of assessing structural reliability a difficult problem. While it is true that the database of strong motion earthquake records across the world has

expanded significantly in recent years, the fact has remained that every new earthquake has brought surprises and, also, that for many parts of the world there exists not even a single recorded near source strong ground motion time history. The latter statement is particularly true if one considers spatial variability characteristics of earthquake loads, in which case, the available database is still smaller. In this context, questions on the estimation of the worst possible scenario for a given structure become relevant. The method of critical excitations has been developed in the existing literature to address this issue. To define the critical excitation, in a given context, one considers the following entities: (a) an engineering structure with specified properties. (b) A class of admissible earthquake excitations, which possess all the known characteristics of a future earthquake. The definition of this class of excitations would be incomplete given that an adequate description for the earthquake load that enables the desired response analysis to be carried out is not available. (c) A measure of structural response, loosely termed as the damage variable, based on the magnitude of which, one can judge the adequacy of the structure to withstand a future earthquake. In this setting, the critical excitation is defined as the member of the class of admissible excitations, which maximizes the specified

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damage variable associated with the given structure. It immediately follows that the critical excitation is dependent on the system characteristics and also on the system response variable chosen for optimization.

Research into the development of critical earthquake load models is now about three decades old. The early studies were due to Drenick [6], Shinozuka [19] and Iyengar [7] and the recent review paper by Takewaki [21] presents a comprehensive account of the current state of the art. The existing studies now cover linear/non-linear systems, single point/multi-point earthquakes, time/frequency domain representations for the loads and deterministic/stochastic models for the ground acceleration. Notwithstanding these developments, questions on the use of critical excitation for engineering design have received little research attention. The key difficulty here lies in the treatment of the intimate link that exists between system properties and the characterization of the critical excitation. In the context of structural design for critical excitation, it is important to note that both the structure and excitation are essentially partially specified. The available properties on the earthquake could include, for example, total average energy, dominant ground frequency, frequency range and non-stationarity trend. Similarly, the information on the structure to be designed could include details, such as, overall structural geometry and a generic probabilistic model for the random variations in the structural parameters. If an iterative strategy for structural design for critical excitations were to be adopted, as is conventionally done in a traditional design process, it would follow that, at each iteration, not only the structural parameters change, but also, the excitation would change. This complicates the design problem considerably. In a recent study, Takewaki [22] has noted some of these difficulties and has formulated a strategy that involves random process models for the earthquake excitation. Here, the design problem is formulated in terms of maximization of a global stiffness parameter subjected to constraints on the power and intensity of earthquake excitations and on total cost. Takewaki [20] studied the problem of optimal placement of dampers to mitigate vibrations in a structure that is loaded by its critical earthquake excitation. An alternative approach based on the use of convex models for loading uncertainties has been developed by Pantilides and Tzan [15] and Tzan and Pantilides [23] for the design of structures with/without active elements under earthquake loads.

In the present study, we explore the problem of design of structures for critical earthquake excitations using reliability based design procedures. The study is an extension of earlier investigations by Manohar and Sarkar [14], Sarkar and Manohar [17,18] and Abbas and Manohar [1,3] on the development of stochastic critical earthquake load models using mathematical programming tools within the framework of random vibration methods. Whereas, the earlier work by Sarkar and Manohar focused on determination of critical excitations that maximize the highest variance of

the structural response, the more recent study by Abbas and Manohar has developed critical excitations which maximize probability of failure of the structure with respect to a specified performance function. The later work also takes into account the possibility of structural parameters being random in the determination of critical excitations. These studies have covered both linear and non-linear systems and also the multi-component and spatial variation features of earthquake loads. The focus of the present study is on developing a strategy for structural design for critical earthquake loads within the framework of reliability based design. We consider linear systems subjected to earthquake loads that are modeled as a product of a partially known, zero mean, stationary, Gaussian random process and a known time-dependent modulating function. The information on the stationary part of the excitation is taken to include the variance, average zero crossing rate, frequency range and average rate of entropy; the power spectral density (psd) function of the process itself is taken to be an unknown. The structural parameters are taken to be random with a specified form of the joint probability density function (pdf). The mean of the structural parameters are taken to be the unknown design variables. The coefficient of variation to be expected in the structural parameters is assumed to be specified. The problem on hand consists of determining the structural design parameters and the unknown psd of the input so that the resulting structure–excitation pair possesses a target level of structural reliability against a specified performance criterion. The solution to this problem is obtained by employing concepts from inverse first order reliability method (FORM). Thus, if \mathbf{U} denotes the basic standard normal random variables in a reliability problem, β the reliability index, $g(\mathbf{U})$ the performance function, and $Y(\mathbf{U})$ the structural capacity, the basic problem of inverse reliability can be stated as finding $Y_{\text{cap}} = \max Y(\mathbf{U})$ subject to the constraint $|\mathbf{U}| = \beta_0$ where β_0 is the target reliability. The method essentially searches a hypersphere of constant radius β_0 to find the maximum response. The structural capacity so determined would thus be calibrated to a specified reliability level. The concepts of inverse FORM have been developed in the existing literature by Winterstein et al. [26], Maes and Huyse [13], Der Kiureghian and Neuenhofer [10], Li and Foschi [11], Lindt and Niedzwecki [12] and Bhattacharya et al. [4]. These studies cover various issues including computational details, treatment of alternative sources of uncertainties and applications to fields of offshore structures and earthquake engineering. It is to be noted that the inverse FORM, as has been discussed in the existing literature, is essentially applicable to time invariant structural problems involving random variable models for the system uncertainties. In the present study, since we are dealing with dynamical systems subjected to partially specified random excitations, the application of the inverse FORM requires newer generalizations. The proposed methods are illustrated with respect to singly supported and doubly supported linear

systems. The possibility of spatial variation of earthquake loads is also included in the analysis.

2. Linear systems under single point seismic excitations

2.1. Problem formulation

A N degree of freedom (dof) shear building model subject to earthquake ground acceleration $\ddot{x}_g(t)$ is considered (Fig. 1). The equation of motion governing the floor displacements relative to the ground, for this system, is well-known to be given by

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = -M\{1\}\ddot{x}_g(t) \quad (1)$$

Here M , C and K are, respectively, the structure mass, damping and stiffness matrices of size $N \times N$, and $\{1\} = a$ $N \times 1$ column vector of ones. The ground acceleration $\ddot{x}_g(t)$ is modeled as a zero mean, non-stationary, Gaussian random process of the form

$$\ddot{x}_g(t) = e(t)\ddot{w}_g(t) \quad (2)$$

where $e(t)$ is a specified deterministic envelop function which is taken to be of the form

$$e(t) = A_0[\exp(-\alpha_1 t) - \exp(-\alpha_2 t)]; \quad \alpha_2 > \alpha_1 > 0 \quad (3)$$

$\ddot{w}_g(t)$ is a zero mean stationary Gaussian random process, which is taken to be only partially specified. Based on the earlier investigation carried out by Manohar and Sarkar [14] and Abbas and Manohar [1] it is assumed that the information available on $\ddot{w}_g(t)$ includes (a) the variance (E_0), (b) average rate of zero crossing (n_0^+), and (c) average rate of entropy measured as an increment with reference to a baseline white noise process. These three quantities, respectively, are associated with the total average energy

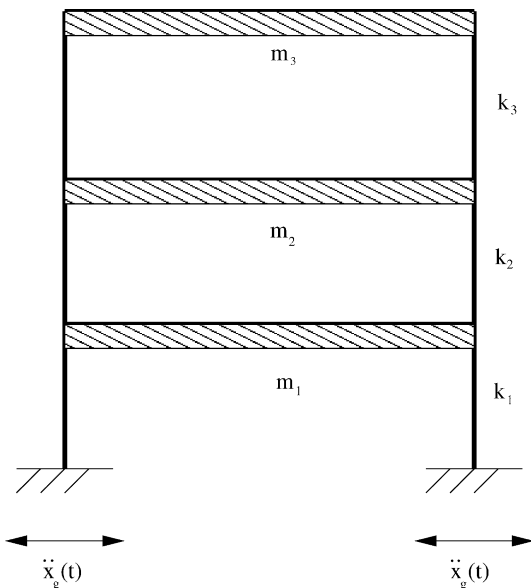


Fig. 1. A shear building model subjected to earthquake ground acceleration.

in the input, the dominant soil frequency and a quantitative measure of disorder that one might expect in recorded earthquake motions. The basic premise of the present study is that these three quantities, together with the definition of deterministic envelop function, can be determined for a given site based upon the set of past recorded earthquake ground motions for the given site or in geologically similar sites. Added to this, we also assume that the frequency range over which the ground motion has significant energy is known in advance. This assumption is again considered realistic, since, it is well known that the frequency range of interest in the ground motion is typically from 0 to about 35 Hz. Notwithstanding the availability of these information, it is important to note that the one sided power spectral density (psd) function, $S(\omega)$, of the stationary component, $\ddot{w}_g(t)$ is taken to be not known *a priori*.

The meaning of the quantities E_0 and n_0^+ are well known. The quantification of disorder to be expected in an earthquake signal has been discussed in the earlier works of Manohar and Sarkar [14] and Abbas and Manohar [1]. In this context, it may be recalled that, for a zero mean, band limited, stationary Gaussian random process $\xi(t)$ with PSD function $S(\omega)$, the average entropy rate is given by [16]

$$\bar{H} = \log_e \sqrt{2\pi e} + \frac{1}{2(\omega_2 - \omega_1)} \int_{\omega_1}^{\omega_2} \log_e S(\omega) d\omega \quad (4)$$

Here (ω_1, ω_2) is the frequency bandwidth. One would face serious computational difficulties in computing \bar{H} , if $S(\omega)$ approaches zero for some ω in (ω_1, ω_2) . To circumvent this difficulty, one can define a reference white noise with intensity I and calculate the change in the entropy rate of $\xi(t)$ when this white noise is added to $\xi(t)$. It can be shown that this increase in the entropy rate is given by

$$\Delta \bar{H} = \int_{\omega_1}^{\omega_2} \log_e \left[1 + \frac{S(\omega)}{I} \right] d\omega \quad (5)$$

The idea of imposing a constraint on $\Delta \bar{H}$ in the determination of critical earthquake excitations has been studied by Abbas and Manohar [1]. These authors have noted that, in the absence of these constraints, the resulting critical excitations become nearly deterministic in nature and produce conservative estimates of critical response. The near deterministic nature of the critical excitations here is manifest in the form of the critical PSD functions being highly narrow banded with average power getting concentrated predominantly at the fundamental system natural frequency. The associated samples of excitation time histories would be poor in frequency content and display nearly sinusoidal behavior. Such 'ordered' excitations would hardly serve as valid models for earthquake inputs. This points towards the fact that the prior knowledge of earthquake records being random in nature has not been made adequate use of in defining the critical excitations. The notion of entropy rate overcomes this limitation and this notion here essentially enables the quantification of amount

of disorder that one might expect in a realistic earthquake load. It may also be noted in this context that entropy based principles offer powerful modeling tools in the treatment of random phenomena with incompletely specified probability space: see, for example, the monograph by Kapur [8] on this topic. Given the fact that the random critical excitation models essentially deal with incompletely specified random processes, it is reasonable to expect that the concept of entropy rate has bearing on the development of these models.

To proceed further, the stationary process, $\dot{w}_g(t)$ in Eq. (2) is represented as

$$\dot{w}_g(t) = \sum_{n=1}^{N_f} (A_n \cos \omega_n t + B_n \sin \omega_n t) \quad (6)$$

Here, the frequencies ω_n , $n=1,2,\dots,N_f$, are selected to span satisfactorily the ground acceleration frequency range (ω_0, ω_c) . The coefficients A_n , B_n are modeled as a vector of zero mean Gaussian random variables that satisfy the conditions

$$\langle A_m A_n \rangle = \langle B_m B_n \rangle = \sigma_n^2 \delta_{mn}; \quad \langle A_m B_n \rangle = 0 \quad \forall m, n = 1, 2, \dots, N_f \quad (7)$$

Here, $\langle \cdot \rangle$ denotes mathematical expectation operator, δ_{mn} is the Kronecker delta and σ_n^2 are given by

$$\sigma_n^2 = \int_{\omega_n}^{\omega_{n+1}} S(\omega) d\omega \quad \forall n = 1, 2, \dots, N_f \quad (8)$$

As is well known, conditions in Eq. (7) are necessary for $\dot{w}_g(t)$ to be a stationary random process. It can be shown that this discretized random process has the auto-covariance and one sided psd functions given, respectively, by

$$\begin{aligned} D_j(t) &= \exp(-\eta_j \omega_j t) [A_j \cos(\omega_{dj} t) + B_j \sin(\omega_{dj} t)] - \sum_{n=1}^{N_f} M_{Dj} A_0 \exp(-\alpha_1 t) \\ &\times \frac{A_n \cos(\omega_n t - \theta_{nj1}) + B_n \sin(\omega_n t - \theta_{nj1})}{\sqrt{(M_{Dj} \alpha_1^2 - M_{Dj} \omega_n^2 - C_{Dj} \alpha_1 + K_{Dj})^2 + (C_{Dj} \omega_n - 2M_{Dj} \alpha_1 \omega_n)^2}} + \sum_{n=1}^{N_f} M_{Dj} A_0 \exp(-\alpha_2 t) \\ &\times \frac{A_n \cos(\omega_n t - \theta_{nj2}) + B_n \sin(\omega_n t - \theta_{nj2})}{\sqrt{(M_{Dj} \alpha_2^2 - M_{Dj} \omega_n^2 - C_{Dj} \alpha_2 + K_{Dj})^2 + (C_{Dj} \omega_n - 2M_{Dj} \alpha_2 \omega_n)^2}} \end{aligned} \quad (14)$$

$$R(\tau) = \sum_{n=1}^{N_f} \sigma_n^2 \cos(\omega_n \tau); \quad G(\omega) = \sum_{n=1}^{N_f} \sigma_n^2 \delta(\omega - \omega_n); \quad (9)$$

The total average energy, E_t , and the average rate of zero crossing, n_0^+ , can be expressed in terms of $e(t)$ and $G(\omega)$, respectively, as

$$E_t = E_0 \int_0^\infty e^2(t) dt; \quad n_0^+ = \frac{1}{2\pi} \sqrt{\frac{E_2}{E_0}}; \quad (10)$$

Here, the quantities E_0 and E_2 are the spectral moments of $G(\omega)$, given by

$$E_k = \int_{\omega_0}^{\omega_c} \omega^k G(\omega) d\omega \quad k = 0, 2 \quad (11)$$

Using Eq. (9), in Eq. (5) one gets

$$\Delta \bar{H}_\omega = \frac{1}{2(\omega_c - \omega_0)} \sum_{n=1}^{N_f} (\omega_n - \omega_{n-1}) \ln \left[1.0 + \frac{\sigma_n^2}{I(\omega_n - \omega_{n-1})} \right] \quad (12)$$

It may be noted that, for a given frequency range (ω_0, ω_c) , and, for a given total average energy, a band limited white noise would possess the highest entropy rate, and, conversely, a narrow band signal would possess the least entropy rate. A realistic ground motion, on the other hand, is unlikely to be an ideal band limited white noise nor an ideal narrow band signal. Consequently, the entropy rate associated with realistic ground motion is expected to be bounded between these two limits.

The relative displacement vector $x(t)$ in Eq. (1) is represented, using modal expansion method, as $\{x(t)\} = [\phi] \{q(t)\}$, where $[\phi]$ is the modal matrix, and $\{q(t)\}$ is the vector of generalized coordinates. It can be shown that the j th mode displacement, q_j , is given by $q_j(t) = \gamma_j D_j(t)$, where γ_j is the participation factor for the j th mode given by

$$\gamma_j = - \frac{\phi_j^T M \{1\}}{\phi_j^T M \phi_j} \quad (13)$$

and D_j is the relative displacement of the j th mode under a support excitation $\ddot{x}_g(t)$. The expression for $D_j(t)$ can be shown to be given by

Here θ_{nj1} and θ_{nj2} are given by

$$\theta_{nj1} = \tan^{-1} \left[\frac{(C_{Dj} \omega_n - 2M_{Dj} \alpha_1 \omega_n)}{(M_{Dj} \alpha_1^2 - M_{Dj} \omega_n^2 - C_{Dj} \alpha_1 + K_{Dj})} \right]$$

$$\theta_{nj2} = \tan^{-1} \left[\frac{(C_{Dj} \omega_n - 2M_{Dj} \alpha_2 \omega_n)}{(M_{Dj} \alpha_2^2 - M_{Dj} \omega_n^2 - C_{Dj} \alpha_2 + K_{Dj})} \right]$$

where η_j , ω_j and ω_{dj} are, respectively, the damping ratio, natural frequency and damped natural frequency of the j th mode. The quantities $M_{Dj} = \phi_j^T M \phi_j$, $C_{Dj} = \phi_j^T C \phi_j$

and $K_{Dj} = \phi_j^T K \phi_j$, respectively, denote the j th modal mass, damping and stiffness. The constants of integration, A_j and B_j , are determined from the specified initial conditions.

The structural floor masses M_i and inter-storey stiffnesses K_i ($i=1,2,\dots,N$) (Fig. 1) are taken to be a set of random variables that are, in general, mutually dependent and non-Gaussian in nature. Using standard transformation techniques, these random variables are transformed to a set of equivalent standard normal random variables, which, we denote by Z_i ($i=1,2,\dots,2N$). For the purpose of illustration, in the present study, we take M_i and K_i ($i=1,2,\dots,N$), to be a set of mutually correlated lognormal random variables, with $\langle M_i \rangle = \mu_{M_i}$, $\langle K_i \rangle = \mu_{K_i}$, $\text{Var}[M_i] = \sigma_{M_i}^2$, and $\text{Var}[K_i] = \sigma_{K_i}^2$. It is also assumed that the correlation coefficient matrix associated with the random variables M_i and K_i ($i=1,2,\dots,N$), is given. The mean quantities μ_{M_i} and μ_{K_i} ($i=1,2,\dots,N$) are treated as the unknown design variables that are assumed to lie within the specified bounds

$$\mu_{M_i}^L \leq \mu_{M_i} \leq \mu_{M_i}^U; \quad \mu_{K_i}^L \leq \mu_{K_i} \leq \mu_{K_i}^U; \quad \forall i = 1, 2, \dots, N \quad (15)$$

The random variables $\{A_n, B_n\}_{n=1}^{N_f}$ are also expressed as standard normal random variables using the transformation $U_n = A_n/\sigma_n$ and $V_n = B_n/\sigma_n$ ($n=1,2,\dots,N_f$). The objective of the design is taken to ensure that structural reliability, against a specific performance criterion, is not less than a prescribed target reliability value. This problem is approached within the framework of the inverse FORM design strategy and, consequently, the target reliability is expressed in terms of a reliability index. For the purpose of illustration, we consider that the performance of the structure is judged by the highest value of the tip relative displacement of the structure.

To formulate the design problem, we begin by noting that the unknowns in the problem are the standard deviations $\{\sigma_n\}_{n=1}^{N_f}$ that define the input psd function, and, the variables $\{\mu_{M_i}, \mu_{K_i}\}_{i=1}^N$. According to the inverse FORM design strategy, the problem on hand consists of determining $\{U_n, V_n\}_{n=1}^{N_f}$, $\{Z_i\}_{i=1}^{2N}$ at the checkpoint and $\{\sigma_n\}_{n=1}^{N_f}$ along with $\{\mu_{M_i}, \mu_{K_i}\}_{i=1}^N$, so that the quantity

$$X_{\max} = \max_{0 < t < T} |x(\{U_n, V_n, \sigma_n\}_{n=1}^{N_f}, \{Z_i\}_{i=1}^{2N}, \{\mu_{M_i}, \mu_{K_i}\}_{i=1}^N, t)| \quad (16)$$

is maximized subject to the constraints

$$\begin{aligned} \sum_{n=1}^{N_f} (U_n^2 + V_n^2) + \sum_{i=1}^{2N} (Z_i^2) &= \beta_0^2; \quad \sum_{n=1}^{N_f} \sigma_n^2 = E_0; \\ \sigma_n &\geq 0; \quad n = 1, 2, \dots, N_f; \quad \sum_{n=1}^{N_f} \sigma_n^2 \omega_n^2 = E_2; \\ \frac{1}{2(\omega_c - \omega_0)} \sum_{n=1}^{N_f} (\omega_n - \omega_{n-1}) \ln \left[1.0 + \frac{\sigma_n^2}{I(\omega_n - \omega_{n-1})} \right] &\geq \Delta \bar{H}_w \\ \mu_{M_i}^L &\leq \mu_{M_i} \leq \mu_{M_i}^U; \quad \mu_{K_i}^L \leq \mu_{K_i} \leq \mu_{K_i}^U; \quad i = 1, 2, \dots, N; \end{aligned} \quad (17)$$

Here, β_0 is the target reliability index that the design must achieve and $\Delta \bar{H}_w$ denotes the specified entropy rate. It may be emphasized here that, in applying inverse FORM in the present context, the structural ‘capacity’ is treated as being deterministic. The spirit of the method essentially consists of searching points lying on the hypersphere of radius β_0 , the point at which the response X_{\max} (that is the ‘demand’) is the highest. If we provide capacity as being equal to this highest demand, then the point on β_0 -hypersphere at which the demand is highest can be construed as the design point. Thus, the performance function, defined as the capacity minus the demand, would be zero at the checkpoint, and on all other points on the β_0 -hypersphere, the function would be greater than zero. That is, all points on the β_0 -hypersphere, excepting the checkpoint, lie in the safe region, while the checkpoint itself lies on the limit surface. Thus, the point on the β_0 -hypersphere at which the response is maximum is indeed the traditional checkpoint encountered in the FORM.

Let us now denote the values of $\{U_n, V_n\}_{n=1}^{N_f}$, $\{Z_i\}_{i=1}^{2N}$ at the checkpoint as $\{U_n^*, V_n^*\}_{n=1}^{N_f}$, $\{Z_i^*\}_{i=1}^{2N}$ and place these quantities into a single vector denoted by

$$Y = [U_1^*, U_2^*, \dots, U_{N_f}^*, V_1^*, V_2^*, \dots, V_{N_f}^*, Z_1^*, Z_2^*, \dots, Z_{2N}^*]^t \quad (18)$$

Thus, the size of vector Y would be $(2N_f + 2N) \times 1$.

Eqs. (16) and (17) constitute a problem in constrained non-linear optimization. The complete solution of the above problem generates the following information.

- (1) The highest relative tip displacement of the structure, X_{\max} , that conforms to the target reliability. The structure must have capacity to withstand this displacement.
- (2) The discrete power spectrum (plot of σ_n^2 versus ω_n) indicating frequency wise average power distribution in the input. This essentially provides the critical psd function model for the input.
- (3) The first $2N_f$ components of Y lead to the definition of a single design earthquake accelerogram given by

$$\ddot{x}_g^*(t) = e(t) \sum_{n=1}^{N_f} \sigma_n (U_n^* \cos \omega_n t + V_n^* \sin \omega_n t) \quad (19)$$

Corresponding to this time history, an associated peak ground acceleration (PGA) can also be identified. The Fourier amplitude spectra of the stationary part of the excitation at the checkpoint can be obtained as

$$W_g^*(\omega) = \sum_{n=1}^{N_f} \sigma_n \sqrt{U_n^{*2} + V_n^{*2}} \delta(\omega - \omega_n) \quad (20)$$

- (4) The variation of $|Y_i/\beta_0|$ versus i , for $i=1,\dots,2N_f+2N$, provides, respectively, the sensitivity of the reliability index with respect to the variables, $U_1, U_2, \dots, U_{N_f}, V_1, V_2, \dots, V_{N_f}, Z_1, Z_2, \dots, Z_{2N}$ at the checkpoint. This plot

helps to rank the variables in decreasing order of their relative importance. It is important to note that this ranking is with respect to the standard normal random variables. Therefore, this information in itself is of little value unless the sensitivities are expressed with respect to the original basic random variables. To clarify this we introduce the notation

$$\theta = \{A_1, A_2, \dots, A_{N_f}, B_1, B_2, \dots, B_{N_f}, M_1, M_2, \dots, M_N, K_1, K_2, \dots, K_N\} \quad (21)$$

In general θ consists of $n^* = 2N_f + 2N$ number of mutually correlated non-Gaussian random variables. Assuming that the knowledge of θ is restricted to the information on the first order pdf of θ_k , $k = 1, 2, \dots, n^*$, and the $n^* \times n^*$ correlation matrix of θ , one can use the Nataf transformations to transform θ into the standard normal space (Y). It can further be shown that

$$\begin{aligned} \frac{\partial \beta_0}{\partial \theta_k^*} &= \sum_{j=1}^{n^*} \frac{\partial \beta_0}{\partial y_j^*} \frac{\partial y_j^*}{\partial \theta_k^*} \\ \frac{\partial \beta_0}{\partial y_j^*} &= - \frac{\sum_{i=1}^{n^*} \left\{ \frac{\partial g}{\partial y_i} \right\}_* \{ \sigma_i \sqrt{\nu_k} \psi_{ik} \}}{\left[\sum_{k=1}^{n^*} \left\{ \sum_{i=1}^{n^*} \left[\frac{\partial g}{\partial y_i} \right]_* [\sigma_i \sqrt{\nu_k} \psi_{ik}] \right\}^2 \right]^{0.5}} \quad (22) \\ \frac{\partial y_j^*}{\partial \theta_k^*} &= \frac{1}{\sqrt{\nu_j}} \frac{\psi_{kj} p_{\theta_k}(\theta_k)}{\phi[\Phi^{-1}\{P_{\theta_k}(\theta_k)\}]} \end{aligned}$$

Here $\{v\}$ and $[\psi]$, respectively, are the vector of eigenvalues and eigenvector matrix of correlation coefficient matrix of θ ; σ_i denotes the standard deviation of θ_i ; $\phi(\cdot)$ and $\Phi(\cdot)$ are, respectively, the first order probability density function and distribution function of a standard normal random variable; as before, a subscript $*$ refers to the value of the associated variable at the checkpoint. Since, in the present problem, $\{A_i, B_i\}_{i=1}^{N_f}$ are mutually uncorrelated random variables, and, also, these random variables are taken to be independent of $\{M_i, K_i\}_{i=1}^N$, the expressions for the sensitivities can be further simplified.

- (5) The determination of optimal values of μ_M and μ_K that constitutes the design of the structure. This would mean that, for these values of the structural design variables, the response produced is the highest and any modification to the design values, would lead to a lower response.
- (6) If σ_n is considered as the nominal value of A_n and B_n , the quantities A_n^*/σ_n and B_n^*/σ_n can be interpreted as the partial safety factors associated with the variables A_n and B_n . For a given n , we introduce the load factor

$$\chi_n = \left(\sqrt{A_n^{*2} + B_n^{*2}} \right) / \sigma_n \quad (23)$$

Similar factors for the variables $\{M_i\}$ and $\{K_i\}$ can also be derived from the known values $\{M_i^*\}$ and $\{K_i^*\}$. These factors for $\{M_i\}$ and $\{K_i\}$ can be defined with respect to the optimal values of μ_M and μ_K .

2.2. Numerical example

For the purpose of illustration, a three storied building frame model as shown in Fig. 1, is considered. The frequency range of the earthquake motion is assumed to lie in the range (0.02, 25.0) Hz. The envelop parameters are taken as $A_0 = 2.17$, $\alpha_1 = 0.13$ and $\alpha_2 = 0.50$, which fix the duration of earthquake to be about 30 s and maximum value of envelop $e(t)$ to unity. The number of frequency terms in the series representation of $G(\omega)$ is taken to be 27 and they are selected in such a fashion that a subset of the ω_n are selected to coincide with the natural frequencies of the mean system and the soil natural frequency. The constraint on the total average energy, E_0 is taken to be $1.45 \text{ m}^2/\text{s}^3$. Depending upon the average zero crossing rate (n_0^+) of the input earthquake signal, E_2 is given by Eq. (10). In estimating average entropy rate, Abbas and Manohar [1] have analyzed a set of 10 earthquake acceleration time histories which are recorded on firm soil sites. Using this data, these authors have fitted a non-stationary Gaussian random process model for the ground acceleration as per the format given in Eq. (2). This has lead to an estimate of $\Delta \bar{H}_w = 0.1003$ (with $I = 0.02 \text{ m}^2/\text{s}^3$). This compares well with $\Delta \bar{H}_w = 0.1004$ that is computed for a Kanai-Tajimi power spectral density function model with soil frequency = $5\pi \text{ rad/s}$, soil damping = 0.60, which correspond to firm ground conditions. In the present study, we have adopted a value of $\Delta \bar{H}_w = 0.09$. The structure is taken to be initially at rest. For the purpose of illustration, $\{K_i\}_{i=1}^N$ and $\{M_i\}_{i=1}^N$ are assumed to be lognormally distributed with coefficient of variation (COV) of 0.05 for both $\{K_i\}$ and $\{M_i\}$ and correlation coefficient of 0.3 between each pair. The respective means of the stiffness and mass parameters are assumed to be given by $3.6 \times 10^8 \leq \mu_{K1} \leq 4.4 \times 10^8 \text{ N/m}$, $2.7 \times 10^8 \leq \mu_{K2} \leq 3.3 \times 10^8 \text{ N/m}$, and $2.7 \times 10^8 \leq \mu_{K3} \leq 3.3 \times 10^8 \text{ N/m}$, $2.7 \times 10^5 \leq \mu_{M1} \leq 3.3 \times 10^5 \text{ kg}$, $2.7 \times 10^5 \leq \mu_{M2} \leq 3.3 \times 10^5 \text{ kg}$ and $2.7 \times 10^5 \leq \mu_{M3} \leq 3.3 \times 10^5 \text{ kg}$. Modal damping ratio of 0.05 is assumed for all the modes.

The optimization problem stated in Eqs. (16) and (17) here is tackled by using the sequential quadratic programming method embedded in the CONSTR program of the MATLAB optimization toolbox [5]. The termination tolerances for the design variables, objective function and constraints violations are, respectively, taken to be 10^{-3} , 10^{-3} and 10^{-3} . To start the optimization solver, it was assumed that

$$\begin{aligned} \{U_i^2\}_{i=1}^{N_f} &= \{V_i^2\}_{i=1}^{N_f} = \{Z_i^2\}_{i=1}^{2N} = \frac{\beta_0^2}{(2N_f + 2N)} \\ \{\sigma_i^2\}_{i=1}^{N_f} &= \frac{E_0}{N_f}; \quad \mu_{M_i} = \mu_{M_i}^L + 0.5(\mu_{M_i}^U - \mu_{M_i}^L); \\ \mu_{K_i} &= \mu_{K_i}^L + 0.5(\mu_{K_i}^U - \mu_{K_i}^L) \end{aligned} \quad (24)$$

Clearly, this starting solution satisfies the first three and the last two sets of constraints listed in Eq. (17), but, in general, it would not satisfy the remaining two sets of

constraints. The success of the optimization solution is, however, not influenced by this feature. Since, the constrained optimization problem on hand is non-linear in nature, questions on whether the optimal solutions obtained are indeed global in nature or not become relevant. While a general answer to this question is difficult to provide, it was, however, verified in the numerical work that for a wide alternative choices for the starting solutions, the same optimal solution was invariably obtained. To gain insight into the nature of critical excitation and the accompanying structural design, parametric studies have been conducted by varying n_0^+ and β_0 . Limited results from these calculations are displayed in Figs. 2 and 3 and Tables 1 and 2. Figs. 2 and 3 show results on critical input psd function, amplitude spectrum of $\ddot{w}_g(t)$ at the checkpoint, ground acceleration time history at the checkpoint, and the partial safety factors, χ_n for $n_0^+ = 2.5$ Hz and $\beta_0 = 2$ and 4, respectively. Table 1 shows the summary of maximum displacement and the PGA at the checkpoint for different combinations of β_0 and n_0^+ .

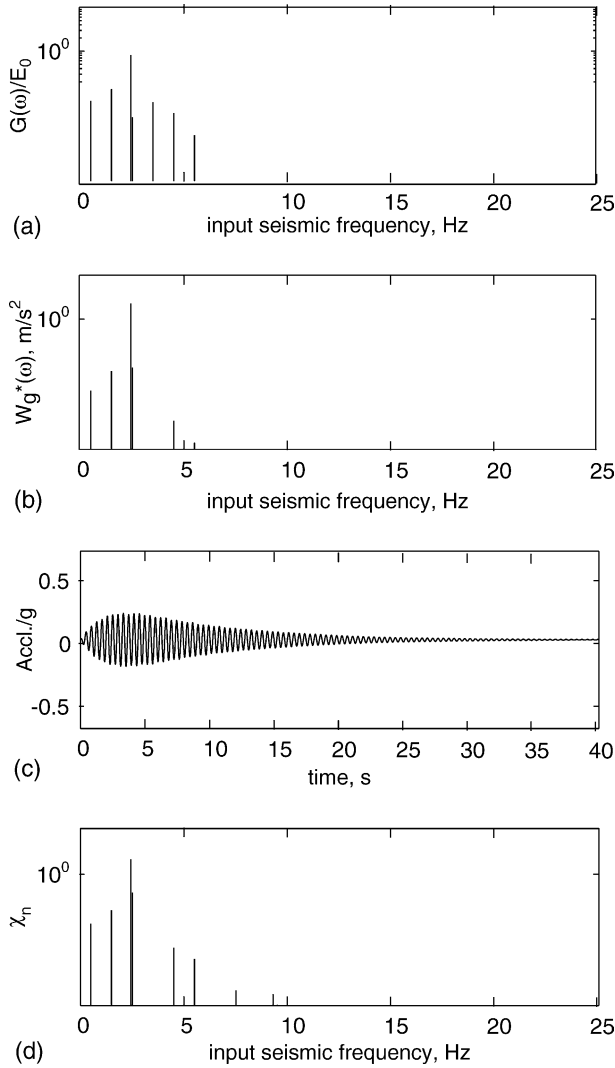


Fig. 2. Example on a 3 dof system under single point excitation, $\beta=2$; (a) critical discrete power spectrum; (b) discrete Fourier spectrum at checkpoint; (c) excitation time history at checkpoint; (d) partial safety factors.

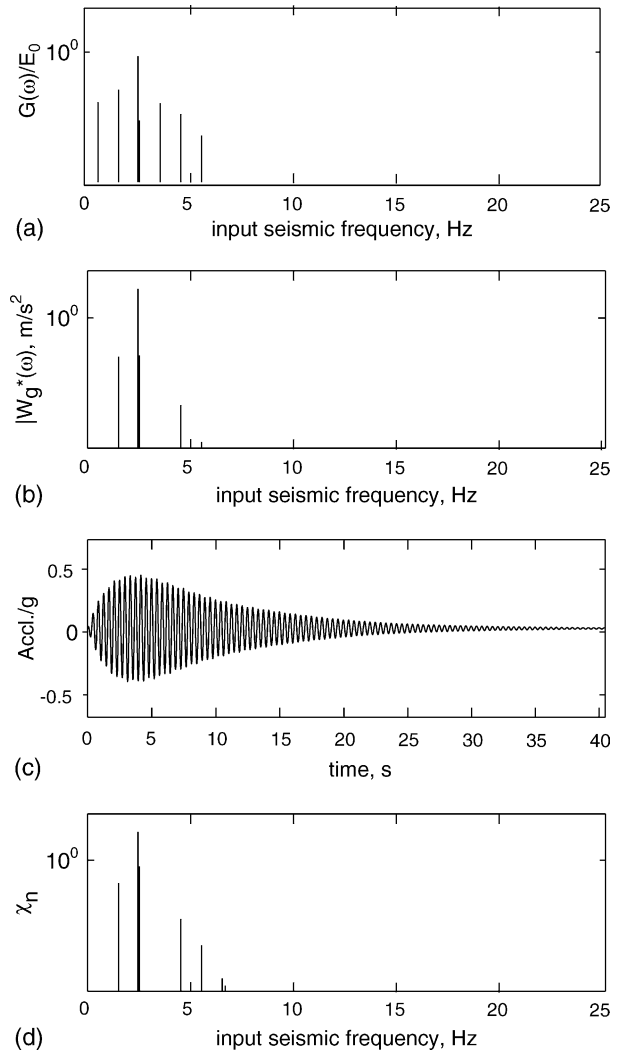


Fig. 3. Example on a 3 dof system under single point excitation, $\beta=4$; (a) critical discrete power spectrum; (b) discrete Fourier spectrum at checkpoint; (c) excitation time history at checkpoint; (d) partial safety factors.

The system natural frequencies at the checkpoint are also shown in this table. Table 2 shows the values of the mass and stiffness parameters at the checkpoint. Computations were also made on the sensitivity factors as given in Eq. (22). Based on the above numerical investigations, the following observations are made.

- (1) From Table 1, it can be observed that, with increase in target reliability index, the response of the system and the PGA at the checkpoint also show increasing trend.

Table 1
Example on a three dof system under single point excitation

β_0	n_0^+ (Hz)	f (Hz)	X_{max} (m)	PGA (g)
2	2.5	2.41, 6.61, 9.25	0.0908	0.209
4	2.5	2.41, 6.61, 9.25	0.1966	0.420
6	2.5	2.41, 6.58, 9.22	0.2655	0.623
3	2.0	2.43, 6.64, 9.29	0.1062	0.274
3	3.5	2.40, 6.60, 9.23	0.1411	0.325
3	5.0	2.44, 6.65, 9.30	0.1410	0.323

Table 2

Example on a three dof system under single point excitation; design value of system parameters

β_0	n_0^+ (Hz)	M_1^*	M_2^*	M_3^*	K_1^*	K_2^*	K_3^*
2	2.5	299912.188	299652.569	299402.414	3.997×10^8	2.997×10^8	2.993×10^8
4	2.5	300842.618	299493.240	298220.911	4.007×10^8	3.003×10^8	2.984×10^8
6	2.5	302340.160	300328.956	298701.095	3.998×10^8	2.998×10^8	2.968×10^8
3	2.0	300729.407	297985.328	295034.079	4.043×10^8	3.022×10^8	2.986×10^8
3	3.5	300091.075	300323.005	300711.830	3.982×10^8	2.989×10^8	2.991×10^8
3	5.0	300894.378	297336.735	293551.710	4.059×10^8	3.031×10^8	2.985×10^8

This is consistent with the expectation that a structure that is designed for higher reliability, would be capable of withstanding higher levels of inputs.

- (2) The effect of increasing β on the critical $G(\omega)/E_0$, amplitude, $|W_g^*(\omega)|$, excitation time history, $\ddot{x}_g^*(t)$, and partial safety factors, χ_n was studied. It was observed that the shape of the critical input psd is unaffected as β is varied. The time history at the checkpoint, however, showed the expected behavior of increase in PGA with β accompanied by relatively minor variation in the frequency content.
- (3) For a fixed value of β , change in value of n_0^+ was seen to produce qualitative changes in the shape of critical psd as well as $|W_g^*(\omega)|$. It was observed that, for n_0^+ less than fundamental system natural frequency, the maximum response produced and the PGA at checkpoint were less than similar results for the case when n_0^+ was greater than the fundamental system natural frequency.
- (4) The study of the sensitivity factors $\partial\beta_0/\partial\theta_k^*$, for $\beta_0=2$ and 4 (Eq. (22)), revealed that for $\beta_0=4$, the first five most influential random variables were $\{A_3, B_4, B_2, B_3, A_2\}$ with $\{\omega_2, \omega_3, \omega_4\} = \{1.5000, 2.4303, 2.5000\} \times 2\pi$ rad/s (Eq. (6)). The sensitivity factors corresponding to these variables were found to be, respectively, $\{0.9768, 0.8733, 0.1292, 0.1231, 0.1049\}$. As one might expect, the sensitivity factors with respect to $\{M_i, K_i\}_{i=1}^N$ were found to be much lower than similar factors corresponding to the excitation variables $\{A_i, B_i\}_{i=1}^{N_f}$. These factors were found to be $\{4.965 \times 10^{-10}, 8.336 \times 10^{-10}, 1.010 \times 10^{-8}, 1.150 \times 10^{-6}, 3.987 \times 10^{-7}, 1.271 \times 10^{-6}\}$, respectively, associated with $\{M_i, K_i\}_{i=1}^3$. The trend in the variation of the sensitivity factors for $\beta_0=2$ was found to be broadly similar.
- (5) The optimal values of μ_{mi} and μ_{ki} were found to be close to $\mu_{mi}=3.0 \times 10^5$ kg and $\mu_{k1}=4.0 \times 10^8$ N/m, $\mu_{k2}=\mu_{k3}=4.0 \times 10^8$ N/m and these optimal values were largely insensitive to changes in n_0^+ and β_0 .

3. Generalization to multi-supports excitations

The important effect that the spatial variability of the earthquake load has on spatially extended structures, has been well recognized in the existing literature in the recent decades (see, for example, Refs. [9,24,25,27–29]).

Accordingly, engineering models employing spatial variability of earthquake loads have been employed in the seismic response analysis of long span bridges and large dams. Another class of problem in earthquake engineering, wherein the spatial variability of support motions is considered as being important in design, arises in the case of piping structures, housed inside nuclear and petrochemical industrial plants. The works of Sarkar and Manohar [18] and Abbas and Manohar [2] have addressed the problem of developing critical excitation model for structures subjected to partially specified spatially varying ground motion. Given the importance of including spatial variability features of earthquake ground motions in structural response analysis, it is of interest to extend the scope of the formulation presented in the preceding section to include spatial varying earthquake load models.

3.1. Problem formulation

We illustrate this extension with respect to the design of a doubly supported single degree of freedom (sdof) system with the two supports subjected to a vector of zero mean non-stationary Gaussian random accelerations (Fig. 4). Such a system can be viewed as a simple idealization of a long span bridge, a piping in a power plant, or, alternatively, as a generic model representing single mode approximation of a doubly supported spatially extended structure.

Using the notation $x^{\text{Tot}}(t)$ =total displacement of mass M , and $x(t)$ =dynamic displacement given by $x(t)=x^{\text{Tot}}(t)-0.5[x_g(t)+y_g(t)]$, the equation governing $x(t)$ can be shown to

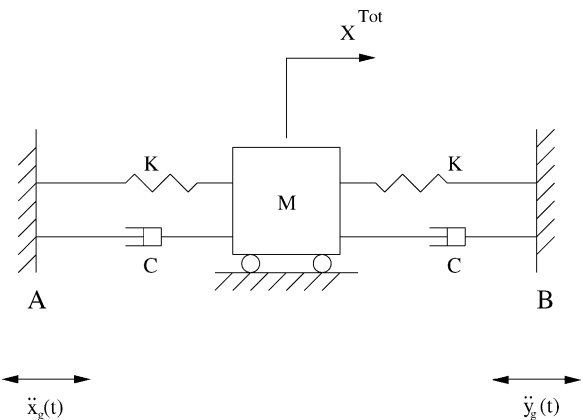


Fig. 4. A doubly supported SDOF system under spatially varying support motions.

be given by

$$M\ddot{x}(t) + \bar{C}\dot{x}(t) + \bar{K}x(t) = -\frac{M}{2}(\ddot{x}_g(t) + \ddot{y}_g(t)) \quad (25)$$

where $\bar{C} = 2C$ and $\bar{K} = 2K$, represent, respectively, the total damping and total stiffness parameters of the structure. To formulate the inverse FORM procedure, we begin by representing the ground accelerations as $\ddot{x}_g(t) = e(t)\ddot{w}_g(t)$ and $\ddot{y}_g(t) = e(t)\ddot{v}_g(t)$, where $e(t)$, as before, is the deterministic envelope function; $\ddot{w}_g(t)$ and $\ddot{v}_g(t)$ are jointly stationary Gaussian random processes with zero mean. We represent these processes as

$$\ddot{w}_g(t) = \sum_{n=1}^{N_f} (A_n \cos \omega_n t + B_n \sin \omega_n t) \quad (26)$$

$$\ddot{v}_g(t) = \sum_{n=1}^{N_f} (C_n \cos \omega_n t + D_n \sin \omega_n t) \quad (27)$$

For the above processes to be jointly stationary, it is necessary that

$$\begin{aligned} \langle A_m A_n \rangle &= \langle B_m B_n \rangle = \sigma_{wn}^2 \delta_{mn}; \\ \langle C_m C_n \rangle &= \langle D_m D_n \rangle = \sigma_{vn}^2 \delta_{mn}; \end{aligned} \quad (28)$$

$$\langle A_m B_n \rangle = \langle C_m D_n \rangle = 0 \quad \forall m, n = 1, 2, \dots, N_f$$

and

$$\begin{aligned} \langle A_m C_n \rangle &= \langle B_m D_n \rangle = \sigma_{acn} \delta_{mn}; \\ \langle A_m D_n \rangle &= -\langle B_m C_n \rangle = \sigma_{adn} \delta_{mn}; \quad \forall m, n = 1, 2, \dots, N_f \end{aligned} \quad (29)$$

Accordingly, the auto-covariance and cross-covariance functions of the processes $\ddot{w}_g(t)$ and $\ddot{v}_g(t)$ can be shown to be given by

$$\begin{aligned} R_{ww}(\tau) &= \sum_{n=1}^{N_f} \sigma_{wn}^2 \cos \omega_n \tau; \\ R_{vv}(\tau) &= \sum_{n=1}^{N_f} \sigma_{vn}^2 \cos \omega_n \tau; \\ R_{wv}(\tau) &= \sum_{n=1}^{N_f} \{\sigma_{acn} \cos \omega_n \tau + \sigma_{adn} \sin \omega_n \tau\} \end{aligned} \quad (30)$$

It may be noted that the covariance matrix of A_n, B_n, C_n and D_n , for a given n , has the form

$$[\bar{C}_n] = \begin{bmatrix} \sigma_{wn}^2 & 0 & \sigma_{acn} & \sigma_{adn} \\ 0 & \sigma_{wn}^2 & -\sigma_{adn} & \sigma_{acn} \\ \sigma_{acn} & -\sigma_{adn} & \sigma_{vn}^2 & 0 \\ \sigma_{adn} & \sigma_{acn} & 0 & \sigma_{vn}^2 \end{bmatrix}$$

In implementing inverse FORM, the random variables A_n, B_n, C_n and D_n are to be transformed to a set of mutually uncorrelated standard normal random variables that we denote by Z_{n1}, Z_{n2}, Z_{n3} and Z_{n4} . It is well known that $\langle Z_n Z_n^t \rangle = T_n^t [\bar{C}_n] T_n$, where T_n is the 4×4 matrix of the eigenvectors of $[\bar{C}_n]$. The uncertainties in the structural parameters are included by treating M and \bar{K} to be mutually correlated lognormal random variables. Using Nataf's transformation, these random variables are expressed in terms of equivalent standard normal variables denoted by Z_1 and Z_2 , respectively. The means and standard deviations of M and \bar{K} are denoted, respectively, by μ_M, μ_K and σ_M, σ_K . In general, it is assumed that M and \bar{K} are mutually correlated.

The multi-supported dynamical system, described as above, is thus characterized by (a) incompletely specified earthquake inputs that are modeled as a pair of non-stationary Gaussian random processes and (b) structural parameters that are modeled as a vector of mutually correlated non-Gaussian random variables with a specified form of joint pdf. We treat the mean of the random vectors M and \bar{K} to be the design variables. These variables are further, taken to be lying in a specified interval, given by

$$\mu_M^L \leq \mu_M \leq \mu_M^U; \quad \mu_K^L \leq \mu_K \leq \mu_K^U; \quad (31)$$

It is assumed that the second order moments of M and \bar{K} , namely, COVs and correlation coefficient, are known. We consider that the performance of the structure is judged by the highest value of the dynamic displacement of the structure.

According to the inverse FORM design strategy, the problem on hand consists of determining $\{Z_{n1}, Z_{n2}, Z_{n3}, Z_{n4}\}_{n=1}^{N_f}, Z_1, Z_2$ at the checkpoint, $\{\sigma_{wn}\}_{n=1}^{N_f}, \{\sigma_{vn}\}_{n=1}^{N_f}, \{\sigma_{acn}\}_{n=1}^{N_f}, \{\sigma_{adn}\}_{n=1}^{N_f}$, along with μ_M, μ_K , so that the quantity

$$X_{\max} = \max_{0 < t < T} |x(\{Z_{n1}, Z_{n2}, Z_{n3}, Z_{n4}, \sigma_{wn}, \sigma_{vn}\}_{n=1}^{N_f}, Z_1, Z_2, \mu_M, \mu_K, t)| \quad (32)$$

is maximized subject to the constraints

$$\sum_{n=1}^{N_f} (Z_{n1}^2 + Z_{n2}^2 + Z_{n3}^2 + Z_{n4}^2) + (Z_1^2 + Z_2^2) = \beta_0^2;$$

$$\sum_{n=1}^{N_f} \sigma_{wn}^2 = E_{0x}; \quad \sum_{n=1}^{N_f} \sigma_{vn}^2 = E_{0y};$$

$$\sigma_{wn} \geq 0; \quad n = 1, 2, \dots, N_f; \quad \sigma_{vn} \geq 0; \quad n = 1, 2, \dots, N_f;$$

$$\sum_{n=1}^{N_f} \sigma_{wn}^2 \omega_n^2 = E_{2x}; \quad \sum_{n=1}^{N_f} \sigma_{vn}^2 \omega_n^2 = E_{2y};$$

$[\bar{C}_n]$ is positive definite $\forall n = 1, 2, \dots, N_f$;

Table 3
Example on doubly supported sdof system

f (Hz)	X_{\max} (m)	PGA _x (g)	PGA _y (g)	μ_M (kg)	μ_K (N/m)	M^* (kg)	K^* (N/m)
3.38	0.0432	0.238	0.221	1008.222	4.600×10^5	1013.632	4.568×10^5

$n_0^+ = 2.5$ Hz, $\beta = 3$.

$$\frac{1}{2(\omega_c - \omega_0)} \sum_{n=1}^{N_f} (\omega_n - \omega_{n-1}) \times \ln \left[1.0 + \frac{\sigma_n^2}{I(\omega_{vn^2} - \omega_{n-1})} \right] \geq \Delta \bar{H}_w; \quad (33)$$

$$\frac{1}{2(\omega_c - \omega_0)} \sum_{n=1}^{N_f} (\omega_n - \omega_{n-1})$$

$$\times \ln \left[1.0 + \frac{\sigma_n^2}{I(\omega_{vn^2} - \omega_{n-1})} \right] \geq \Delta \bar{H}_v;$$

$$\mu_M^L \leq \mu_M \leq \mu_M^U; \quad \mu_K^L \leq \mu_K \leq \mu_K^U.$$

Here, β_0 is the target reliability index that the design must achieve and $\Delta \bar{H}_w$ and $\Delta \bar{H}_v$ denote, respectively, the specified entropy rates of $\ddot{w}_g(t)$ and $\ddot{v}_g(t)$. It can be shown that the requirement that $[\bar{C}_n]$ is positive definite for all n , is equivalent to the condition $\sigma_{acn}^2 + \sigma_{adn}^2 < \sigma_{vn}^2 + \sigma_{vn}^2 \forall n$. Let us denote the values of $\{Z_{n1}, Z_{n2}, Z_{n3}, Z_{n4}\}_{n=1}^{N_f}$, Z_1 , Z_2 at the checkpoint as $\{Z_{n1}^*, Z_{n2}^*, Z_{n3}^*, Z_{n4}^*\}_{n=1}^{N_f}$, and Z_1^*, Z_2^* , respectively and place these quantities into a single vector denoted by

$$Y = [\{Z_{n1}^*, Z_{n2}^*, Z_{n3}^*, Z_{n4}^*\}_{n=1}^{N_f}, Z_1^*, Z_2^*]^t \quad (34)$$

Thus, the size of vector Y would be $(4N_f + 2) \times 1$.

The optimization problem here is again tackled by using the CONSTR program of the MATLAB platform. The termination tolerances for the design variables, objective function and constraints violations are respectively taken to be 10^{-4} , 10^{-4} and 10^{-6} . The variables of optimization here, are $\{\sigma_{vn}, \sigma_{vn}, \sigma_{acn}$ and $\sigma_{adn}\}_{n=1}^{N_f}$, μ_m and μ_k , $\{Z_{n1}, Z_{n2}, Z_{n3}$ and $Z_{n4}\}_{n=1}^{N_f}$, at checkpoint and Z_1 and Z_2 at the checkpoint. The objective function, as can be seen from Eqs. (32) and (33), does not directly depend upon $\{\sigma_{acn}$ and $\sigma_{adn}\}_{n=1}^{N_f}$, but these variables appear in the specification of constraints. An attempt to solve the constrained non-linear optimization problem using sequential quadratic programming tool did not yield satisfactory results. The difficulties were mainly related to the treatment of the variables $\{\sigma_{acn}$ and $\sigma_{adn}\}_{n=1}^{N_f}$ at the step of transforming the correlated Gaussian random variables $\{A_n, B_n, C_n$ and $D_n\}_{n=1}^{N_f}$ to a set of equivalent standard normal variables. This difficulty was overcome in the present study by combining the sequential quadratic programming tool with a random search tool to solve the problem. Specifically, the variables $\{\sigma_{acn}$ and $\sigma_{adn}\}_{n=1}^{N_f}$, expressed in the form of associated correlation coefficients, were simulated randomly as being uniformly distributed between -1 and 1 .

For each realization of $\{\sigma_{acn}$ and $\sigma_{adn}\}_{n=1}^{N_f}$, the optimization problem was solved by excluding $\{\sigma_{acn}$ and $\sigma_{adn}\}_{n=1}^{N_f}$ from the list of optimization variables. This exercise was repeated for N^* numbers of realizations of $\{\sigma_{acn}$ and $\sigma_{adn}\}_{n=1}^{N_f}$. This, in turn, enabled us to identify the optimal $\{\sigma_{acn}$ and σ_{adn} that led to highest response among the ensemble of maximum responses produced.

3.2. Numerical example and discussion

The frequency range of the earthquake motion is assumed to lie in the range (0.02, 15.0) Hz. The envelop parameters are taken as $A_0 = 2.17$, $\alpha_1 = 0.13$ and $\alpha_2 = 0.50$. The number of

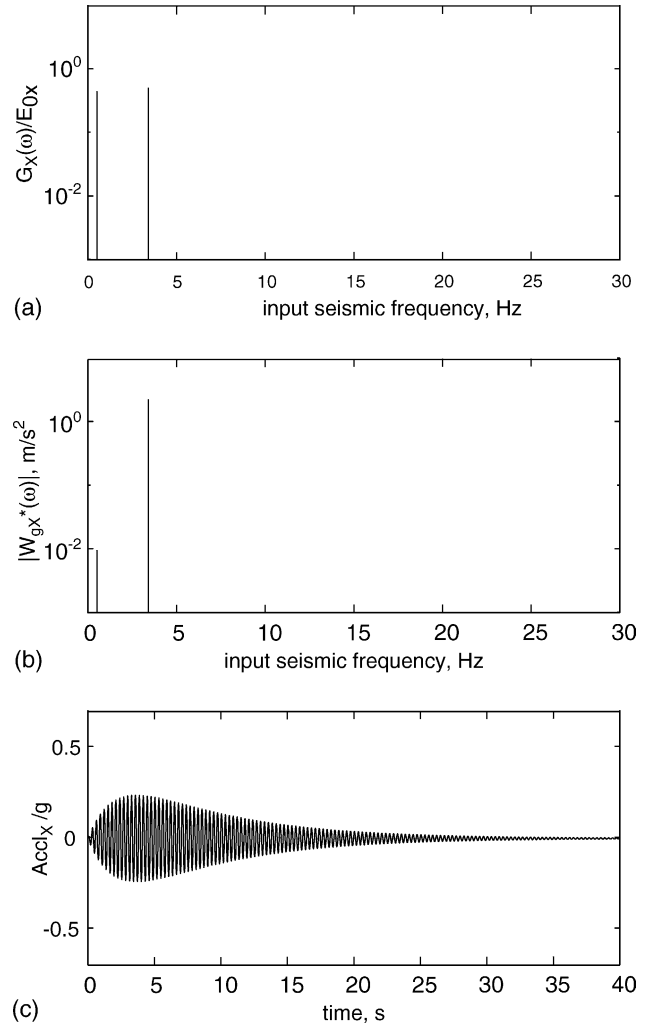


Fig. 5. Example on a doubly supported sdof system; $n_0^+ = 2.5$ Hz, $\beta = 3$; support A: (a) critical auto-discrete power spectrum, (b) amplitude spectrum at checkpoint, (c) excitation time history at checkpoint.

frequency terms in the series representation of $G(\omega)$ is taken to be 15 and their selection is done in such a fashion that a subset of the ω_n are selected to coincide with the natural frequency of the mean system and the soil natural frequency. The constraint on the total average energy of the excitations at the two supports are respectively taken as, $E_{0x} = 1.45 \text{ m}^2/\text{s}^3$ and $E_{0y} = 1.25 \text{ m}^2/\text{s}^3$. The average zero crossing rate (n_0^+) of the input earthquake signals is taken to be 2.5 Hz. $\Delta\bar{H}_w$ and $\Delta\bar{H}_v$ are both taken here, to be 0.09. The structure is assumed to be initially at rest. For numerical illustration, N^* is taken here as 25. It is assumed that $4.37 \times 10^5 \leq \mu_K \leq 4.83 \times 10^5 \text{ N/m}$ and $957.81 \leq \mu_M \leq 1058.63 \text{ kg}$ with COVs of 0.05 for both M and \bar{K} and a correlation coefficient of 0.3 between them.

Table 3 summarizes the results on the natural frequency at the checkpoint, maximum response produces, PGAs, design values of μ_M and μ_K and M^* and K^* . Figs. 5 and 6 show the critical auto-psd function and the excitation characteristics at the checkpoint. The results on partial safety factors are shown in Fig. 7. Fig. 8 shows the spectrum of σ_{acn} and σ_{adn} .

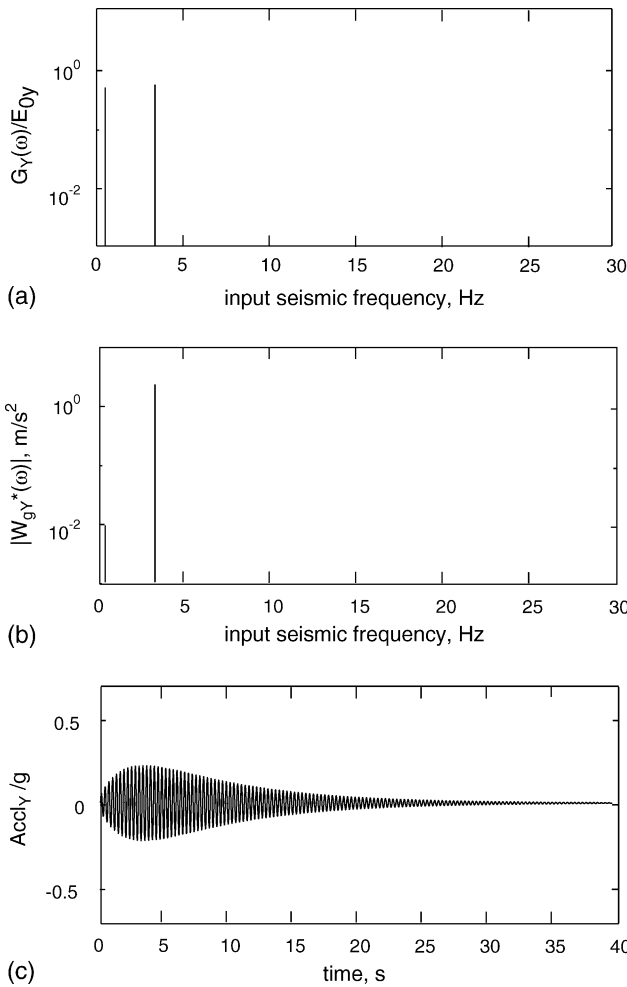


Fig. 6. Example on a doubly supported sdf system; $n_0^+ = 2.5 \text{ Hz}$, $\beta = 3$; support B: (a) critical auto-discrete power spectrum, (b) amplitude spectrum at checkpoint, (c) excitation time history at checkpoint.

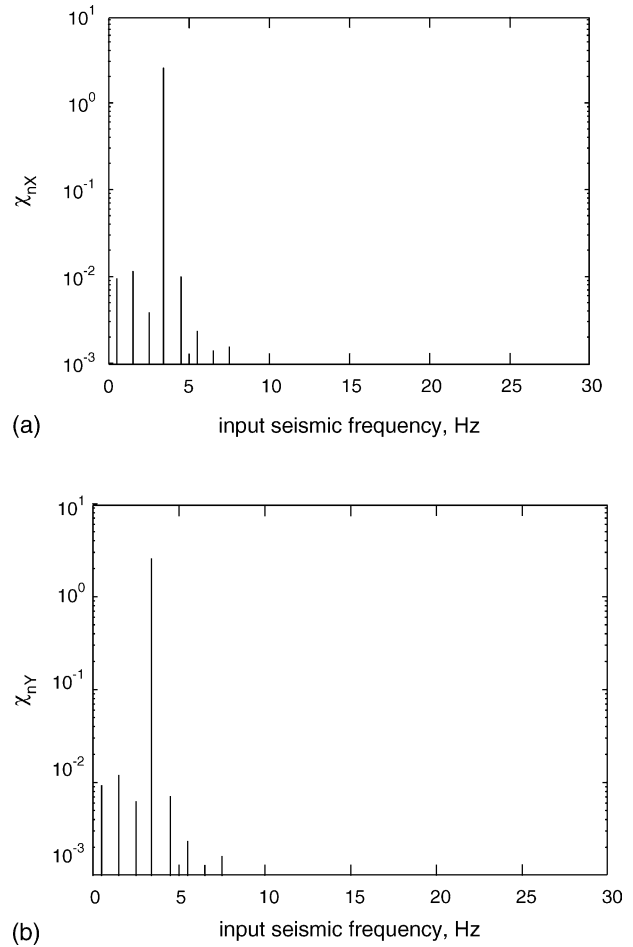


Fig. 7. Example on a doubly supported sdf system; $n_0^+ = 2.5 \text{ Hz}$, $\beta = 3$; (a) partial safety factors for $\ddot{w}_g(t)$, (b) partial safety factors for $\ddot{v}_g(t)$.

Results shown in Figs. 5 and 8, reveal that the critical auto-psd functions and the Fourier amplitude spectra at the checkpoint are nearly identical with dominant part of the average power focused near the system natural frequency. The plots of cross-covariance σ_{acn} and σ_{adn} , also reveal a peak at the structural natural frequency and a peak at the lower end of the frequency spectra. The optimal values of μ_K and μ_M were found to be $\mu_K = 4.60 \times 10^5 \text{ N/m}$ and $\mu_M = 1008.22 \text{ kg}$. The five most influential random variables judged based on the magnitude of $\partial\beta_0/\partial\theta_k^*$ (Eq. (22)) were found to be $\{D_4, B_4, A_4, C_4, D_1\}$ with $\{\omega_1, \omega_4\} = \{0.5, 3.4\} \times 2\pi \text{ rad/s}$ (Eqs. (25) and (26)). The corresponding sensitivity factors were found to be $\{0.6880, 0.5368, 0.4396, 0.0517, 0.0086\}$. The sensitivity factors with respect to M and \bar{K} were found, respectively, to be 6.967×10^{-4} and 1.500×10^{-5} , which are significantly lower than those with respect to the excitation variables.

It may be noted that in applying the proposed method to land based structures such as long span bridges it is possible that a few features of coherence between different support motions could be reliably known. Thus, the structure of the coherence function can reasonably be defined taking into account the distance between the supports and the time lag

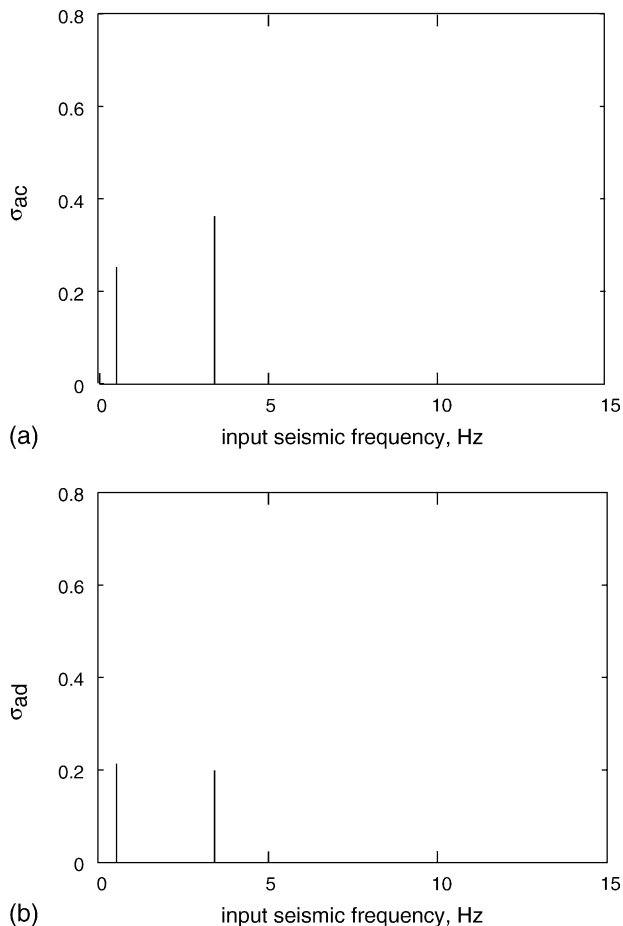


Fig. 8. Example on a doubly supported sdof system; $n_0^+ = 2.5$ Hz, $\beta = 3$; (a) critical σ_{acn} spectrum, (b) critical σ_{adn} spectrum.

effects. In such a case, one can define additional constraints reflecting this prior knowledge on the critical cross-psd functions. This would clearly modify the nature of the critical excitations. However, for multi-supported secondary systems, such as pipings in nuclear power stations, the structure of the coherency functions, based on the knowledge of distance between supports and the time lag of arrival of seismic waves at different supports, are not easy to generalize. Consequently, for this class of problems, we believe that the procedure employed in the present study offers a reasonable framework.

4. Conclusions

The problem of determining critical excitation that produces the highest response in a given structure and, at the same time, satisfies a set of prescribed constraints has been widely studied in the existing literature. The associated problem of design of structures to withstand critical excitations has not received wide attention. The present study explores the application of inverse reliability methods to tackle this class of problems. In the context of structural design for critical excitations, it is important to note that both

the structure and the excitation could be partially specified. The specification of the excitation consists of trustworthy information such as total average energy, non-stationarity trend, frequency trend, dominant soil conditions and measures of disorder. On the other hand, the known features of the structure typically include the form of the joint pdf of the structural parameters and quantitative information on variability characteristics. What is not known about the input could be the psd function and, about the structure, the unknowns could be the nominal values of the structural parameters. The design objective would consist of specifying structural capacity that meets a target reliability. The viewpoint adopted in this study is based upon inverse reliability based design that has been developed in the existing literature in the recent years. The present study has generalized the approach to include transient random loads as well as incompleteness in load specification. The formulation presented is general enough to treat non-stationary random inputs, MDOF systems, uncertainties in specifying the structural parameters and spatial variability of earthquake loads. The solution of the problem provides, as a by product, many useful descriptors of the input and the structural response, such as, definition of a single time history of the input at the checkpoint, measures of sensitivity of various random variables in the problem and partial safety factors associated with the loading and structural parameters. The numerical illustrations are essentially exploratory in nature and have been with respect to simple structures, but at the same time, they are believed to be detailed enough to demonstrate the feasibility of the idea proposed. It is to be emphasized that the present study is based upon the applicability of FORM, and, consequently, it precludes the possibility of existence of multiple design points or multiple points that make comparable contributions to the failure probability. It is clearly desirable to generalize the scope of the present formulations to overcome this limitation. Similarly, further scope exists for improving the efficacy of the solution framework by employing optimization tools such as genetic algorithms into the present formulation. These aspects, however, require further research efforts.

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