

Correlation of densities with shear wave velocities and SPT N values

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Abstract

Site effects primarily depend on the shear modulus of subsurface layers, and this is generally estimated from the measured shear wave velocity (V_s) and assumed density. Very rarely, densities are measured for amplification estimation because drilling and sampling processes are time consuming and expensive. In this study, an attempt has been made to derive the correlation between the density (dry and wet density) and V_s /SPT (standard penetration test) N values using measured data. A total of 354 measured V_s and density data sets and 364 SPT N value and density data sets from 23 boreholes have been used in the study. Separate relations have been developed for all soil types as well as fine-grained and coarse-grained soil types. The correlations developed for bulk density were compared with the available data and it was found that the proposed relation matched well with the existing data. A graphical comparison and validation based on the consistency ratio and cumulative frequency curves was performed and the newly developed relations were found to demonstrate good prediction performance. An attempt has also been made to propose a relation between the bulk density and shear wave velocity applicable for a wide range of soil and rock by considering data from this study as well as that of previous studies. These correlations will be useful for predicting the density (bulk and dry) of sites having measured the shear wave velocity and SPT N values.

Keywords: density, SPT N value, shear wave velocity, correlation

(Some figures may appear in colour only in the online journal)

1. Introduction

The nature of earthquake motion and its effect on structures is significantly affected by the site-specific subsoil profile. The subsoil affects the movement of seismic waves through the ground. Studies have shown that in areas surrounding the epicenter of an earthquake, damage mainly depends on the localized conditions and the damage caused by far-field earthquakes is due to amplification caused by the soil layers. The soil layers change the character of ground movement by amplifying the seismic waves. The shear modulus and shear properties of soil have an important bearing on the extent of any amplification. Therefore, the shear modulus and shear properties need to be determined as a part of site response

studies. During the determination of the shear modulus, the shear wave velocity is obtained through seismic surface wave methods, like the multichannel analysis of surface wave (MASW) or the spectral analysis of surface wave (SASW) methods, and the density is suitably assumed based on the velocity in order to avoid the cumbersome task of drilling boreholes. Until now, a very limited attempt has been made to achieve a reliable model/correlation for obtaining the density from the shear wave velocity and standard penetration test (SPT) N value. Therefore, the present study focuses on the development of correlations between the density and shear wave velocity and the SPT N value using the measured data. Thus, the *in situ* density can be predicted from the measured values of shear wave velocity (V_s) and SPT N value. A relation

has been developed between the bulk density and shear wave velocity for all stratum, i.e. soil and rock, by adding the rock data from Boore (2007) to the data used in the present study. The proposed relation for all soil and rock has been compared to the Gardner relation suggested by Boore (2007).

Several studies were conducted in order to relate the SPT N value and shear wave velocity to various soil properties. The SPT N value has been linked with many properties like the relative density, the consistency, Young's modulus, the shear wave velocity, the shear modulus, the over consolidation ratio and the California bearing ratio (CBR). Several researchers have investigated the relationship between the SPT N value and the relative density (D_r). Meyerhof (1957) conducted laboratory studies and proposed a correlation between the SPT N value and D_r for clean sands, in which the penetration resistance was assumed to increase with the square of the relative density and be in direct proportion to the effective overburden pressure of sand. In essence, Meyerhof's expression specified that the ratio between the normalized SPT N value and the square of the relative density, N_1/D_r^2 , was fixed at 41. It was recognized that penetration resistance depends on the grain size of the soils, that fines-containing sands have a lower SPT blow count than clean sands, and that the ratio gradually decreases with the decreasing mean grain size or its increasing fines content. Gibbs and Holtz (1957) studied the relation between penetration resistance and D_r for fine and coarse sands. A plot was prepared between the penetration resistance and relative density for varying overburden pressures. Data compiled by Fujita (1968) considered the energy transfer ratio during the hammer dropping to be approximately equal to 80%. Tatsuoka *et al* (1978) examined the accuracy of the original Meyerhof expression by using the results of an SPT on normally consolidated sandy deposits along with the *in situ* relative densities of conventional undisturbed samples. It was found that the correlation given by Meyerhof (1957) tends to underestimate the relative density of fine and silty sands. Furthermore, to account for grain size characteristics and fines content, Cubrinovski and Ishihara (1999) introduced the parameter $e_{\max}-e_{\min}$. The authors compiled data points relating this parameter and the ratio between the SPT N value and D_r . A relation was proposed for the SPT N value based on $e_{\max}-e_{\min}$ and D_r , such that it would account for grain size characteristics and fines content. It can be noted here that most of the studies were limited to relating soil properties to relative density based on laboratory samples, and that limited study has been undertaken to relate the *in situ* density with the shear wave velocity or SPT N values. Relative densities are useful for laboratory-based studies, the bulk or dry densities required for numerical geotechnical analysis, site response analysis and the simulation of ground motion. Considerable amounts of work have been done to relate compression wave velocity to the density of rocks. Gardner *et al* (1974) found an empirical relationship between the density and velocity from a series of controlled field and laboratory measurements of brine-saturated rocks (excluding evaporites) from various locations and depths, given by $\rho = aV^m$, where ρ is the density and V is the P-wave velocity. The default values for a and m were given as 0.31 and 0.25, respectively, for the density in g cm^{-3} and

P-wave velocity in m s^{-1} . Studies conducted by Boore (2007) indicated that the relation given by Gardner *et al* (1974) was only applicable for velocity values lying above 1524 m s^{-1} . A plot of density and shear wave velocity both measured in quaternary sediments (derived from Brocher 2005) and from Gardner's relation was prepared, and it was concluded that Gardner's relation could not be used for unsaturated sediments near the surface. The ratio of bulk density for dry and fully saturated rocks was plotted as a function of porosity and using porosity values from 0.2 to 0.35, and the densities were found to be in agreement with those values obtained in quaternary sediments. Boore (2007) also suggested a model including three relations relating density and P-wave velocity: for P-wave velocities less than 1.5 km s^{-1} , for velocities lying between 1.5 km s^{-1} and 6 km s^{-1} and for values greater than 6 km s^{-1} . Another model was suggested for the shear wave velocity also including relations for velocities less than 0.3 km s^{-1} , values lying between 0.3 km s^{-1} and 3.55 km s^{-1} and values greater than 3.55 km s^{-1} . Kim *et al* (2001) suggested a method for the evaluation of density in layer compaction using SASW tests. The authors proposed an evaluation of the shear wave velocity using the SASW test and then a test of the samples obtained from a site using a free-free resonant column test. The effect of confinement was eliminated by normalizing the shear wave velocity. A relationship was developed between the shear wave velocity and density along with a systematic procedure to determine field density. The method proposed was only applicable for non-plastic soils, which are uniformly compacted up to a depth of 4 m. Field applicability was tested at the Hoengsung road construction site and the procedure adopted was found to be valid. Lindseth (1979) derived an empirical relation between the velocity and impedance of rocks, and thereby expressed density as a function of velocity and impedance. Ayres and Theilen (1999) reported that the S-wave velocity of unconsolidated marine sediments showed a good relationship to bulk density, clay content, and water content. Foti and Lancellotta (2004) proposed an approximate equation that explained porosity as a function of compression and shear wave velocity, using the velocity data reported by Hunter (2003). Inazaki and Nagasawa (1991) found that the S-wave velocity, obtained using the PS suspension logging tool measured at five boreholes adjacent each other, had a linear relationship to the mean grain size, solidity (the complement of porosity), an association with reference to lithofacies, and a depositional age. The S-wave velocity also showed a good relation to solidity and density. The majority of the above-mentioned studies were based on laboratory experiments and model study. It is clear that very limited studies were carried out by relating V_s and SPT N values measured *in situ* with the *in situ* density for various materials. The present study is the first of its kind which aims to develop correlations between the measured *in situ* density, the shear wave velocity and the SPT N value. In the present study, subsoil investigation carried out in the city of Lucknow was used to generate a correlation between the density, shear wave velocity and the SPT N value. A total of 354 shear wave velocity and density data sets, and 364 SPT N value and density data sets were used in the process. The overall data was segregated into fine-grained

and coarse-grained soil based on engineering classification, and correlations were developed for all data, fine-grained data and coarse-grained data, separately. The correlations developed for bulk density were compared with Inazaki's (2006) correlations, and the newly developed relations were found to demonstrate good prediction performance. It can be noted here that correlations serve as a quality assurance check in determining test results. The use of correlations is recommended either when specific data is simply not available, when a limited amount of data for the specific property of interest is available, or the validity of certain data is in question (FHWA-IF-02-034 2002). In addition, one must bear in mind that correlations cannot be used as a substitute for soil investigation; they can, however, be used as a guide to authenticate the values obtained from soil investigation.

2. Study area and field testing

In the present work, subsoil investigation conducted in the city of Lucknow for seismic microzonation has been used. Lucknow is the capital of Uttar Pradesh, the most populous state of India, and is situated on the Northern Gangetic plains. The geographical location of Lucknow is between 26.50° North and 80.50° East. The study area is laid with thick alluvial sediments deposited due to river-associated erosion in the Himalayas. Lucknow lies on the bank of the river Gomati from Husainabad to the Dilkusha garden. Regional soil deposits mainly comprise both older and younger alluvium of Ganga–Ghagra interfluvies (GSI 2001). The older alluvium spreads over the vast area between elevations of 115 and 129 m, covering areas such as Chowk, Aminabad, Charbagh and Kakori. Sand mounds 4–5 m in height from ground level have been observed in the Malihabad and Gosainganj areas. Anbazhagan *et al* (2013) presented a detailed discussion on soil deposits and field experiments in the study area. In this study, 22 shear wave velocity profiles and 23 sets of borehole data with SPT N values were identified and selected for developing density correlations. The shear wave velocity was measured using the MASW survey. The MASW test was carried out using a 24-channel geode seismograph in combination with 24 vertical geophones with a frequency of 4.5 Hz. An impulse was generated by striking a 15 pound sledge-hammer against a 30 cm × 30 cm steel plate, generating the surface waves. A geophone interval of 1 m and varying shot distances of 5, 10, 15, 20, and 25 m with 10 stacks were used to reach the maximum penetration depth. More discussion about MASW testing and results can be found in Anbazhagan *et al* (2013). The shear wave velocity values were recorded in the range 100–650 m s⁻¹. Figures 1(a)–(c) show the shear wave velocity profiles from the MASW testing used in the study. Twenty-three boreholes were drilled in the study area, the SPT N was measured and soil samples were also collected as per IS:2131 (1981). All boreholes were drilled with a diameter of 150 mm, as per IS:1892 (1979), and the SPT N values were measured regularly at 1.5 m intervals as per IS:2131 (1981). Disturbed and undisturbed samples were collected at various possible depths as per IS:2132 (1986). The physical properties were measured in the laboratory using the disturbed soil samples

as per IS:1498 (1970) and then used for soil classification. The *in situ* dry and bulk density of the soil was determined as per IS:2720 (Part XXIX) (1975), as presented in the next section. The SPT N values and soil profiles were recorded in the field. The SPT N values recorded ranged from 3 to 50. A plot of the SPT N and depth for the 23 boreholes used in this study is shown in figures 2(a)–(d). More discussion about the SPT N values and V_s values, and a comparison and correlation between the SPT N and V_s can be found in Anbazhagan *et al* (2013). The general soil profile encountered varied from silty sand to clay of low plasticity. The overall data from each borehole was segregated into fine-grained and coarse-grained soils, based on a standard classification system as per IS:1498 (1970)—which is similar to ASTM D2487-06. The classification of the soil present at the site was done based on the percentage weight passing through standard sieves and Atterberg limits. According to IS:1498 (1970), coarse-grained soils are subdivided into gravel and sand based on the percentage of coarse fraction, whereas fine-grained soils are subdivided into three depending on the value of the liquid limit. Inorganic clay of low plasticity (CL), inorganic silts of none to low plasticity (ML), inorganic clays of medium plasticity (CI), inorganic silts of medium plasticity (MI) and silty clay (CL-ML) are grouped as fine-grained soils. Silty sands (SM), poorly graded sands (SP) and poorly graded sand with silt (SM-SP) have been classified as coarse-grained soils. Finally, this data was grouped as fine-grained data, coarse-grained data and all data (all soil types), and was further used to develop the correlation separately.

2.1. Bulk and dry density

During the drilling for the SPT N value measurements, the undisturbed samples were obtained at regular intervals, as per IS:2132 (1986). The samples were collected by driving a thin-walled casing, and the hole was cleaned such that the sample remained undisturbed. The depth of the casing below ground level and the depth of the water table were noted. The assembled sampling tube was inserted and the depth of the bottom of the borehole below ground level, the amount of penetration of the sampling tube and the water level in the borehole was recorded. The sampling tube was pushed in a continuous and rapid motion beyond the casing bottom. By repeating the sampling procedures, samples were taken at every change in the stratum, or at intervals of not more than 1.5 m—which ever was less. This procedure is similar to ASTM D1587-12. The samples for ascertaining the density were obtained as per the drive cylinder method described earlier. The *in situ* dry and bulk density of soil can be determined by the core cutter method as per IS:2720 (Part XXIX) (1975). A cylindrical core cutter (length, 130 mm; internal diameter, 100 mm; wall thickness, 3 mm), beveled at one end was used. The internal volume of the core-cutter in cubic centimeters was calculated from its dimensions measured to the nearest 0.25 mm, and the cutter was weighed to the nearest gram. A small area—approximately 30 cm² of the soil layer to be tested—was exposed and leveled. A steel dolly 2.5 cm high and with a 10 cm internal diameter was placed on top of the cutter, and

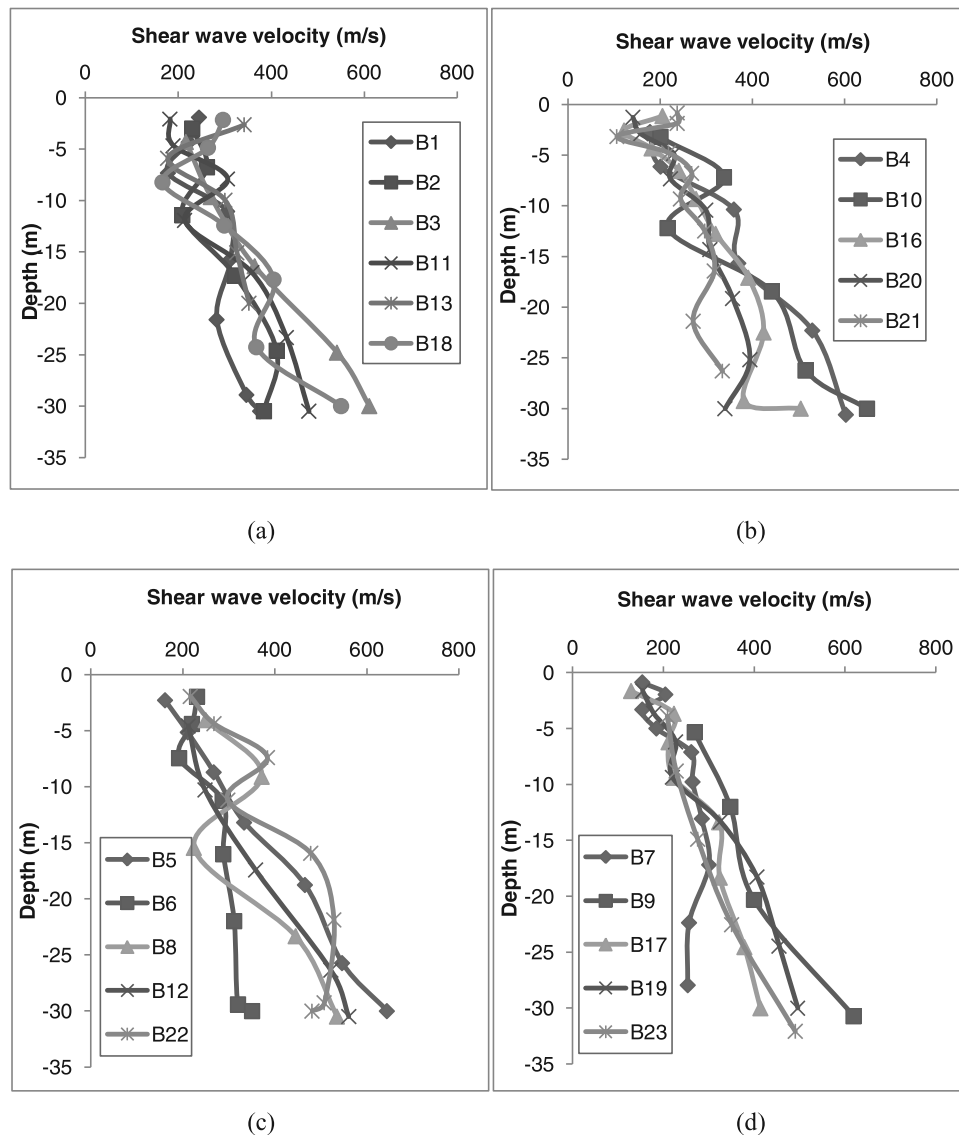


Figure 1. (a)–(d) Plot of shear wave velocity versus depth of data considered in the study.

the latter was rammed down vertically into the soil layer until only about 15 mm of the dolly protruded above the surface. The cutter was dug out of the surrounding soil and the ends of the soil core were then trimmed flat to the ends of the cutter by means of a straight edge. The cutter containing the soil core was weighed to the nearest gram. The soil core was removed from the cutter, a representative sample was placed in an airtight container, and its water content was determined as per IS:2720 (part II) (2010). The bulk density was calculated from the recorded weight and volume. The dry density was then calculated from the bulk density and water content. The variations of wet and dry density and depth for the 23 borehole locations are shown in figures 3(a)–(c) and 4(a)–(c), respectively. The bulk density values used varied from $1.5\text{--}2.25\text{ g cc}^{-1}$, whereas the dry density ranged from $1.38\text{--}1.85\text{ g cc}^{-1}$. The general assumption that density increases with depth was satisfied by the increasing trend shown by the majority of the boreholes. However, it can also be observed that a few densities are lower, which may be due to the deposition process and age of the deposit. These bulk densities and dry density

values are used to generate the correlation between the SPT N and V_s values.

3. New correlations

Empirical corrections are a part of geotechnical engineering and are widely used in many design applications, and several such empirical correlations are available in the literature. The Novo Tech software (2014) summarizes most of the widely used geotechnical correlations. Several correlations are available for laboratory/field-measured engineering properties and field-measured engineering properties. Of these, the correlations between the SPT N and the V_s /shear modulus are the most popular and frequently used in earthquake geotechnical engineering (Anbazhagan *et al* 2012). At the same time, very limited correlations are available for laboratory/field-measured index properties and field-measured engineering properties, such as the SPT N or V_s . In this section, the measured bulk density and dry density data has been correlated with the shear wave velocity and SPT N values. The general

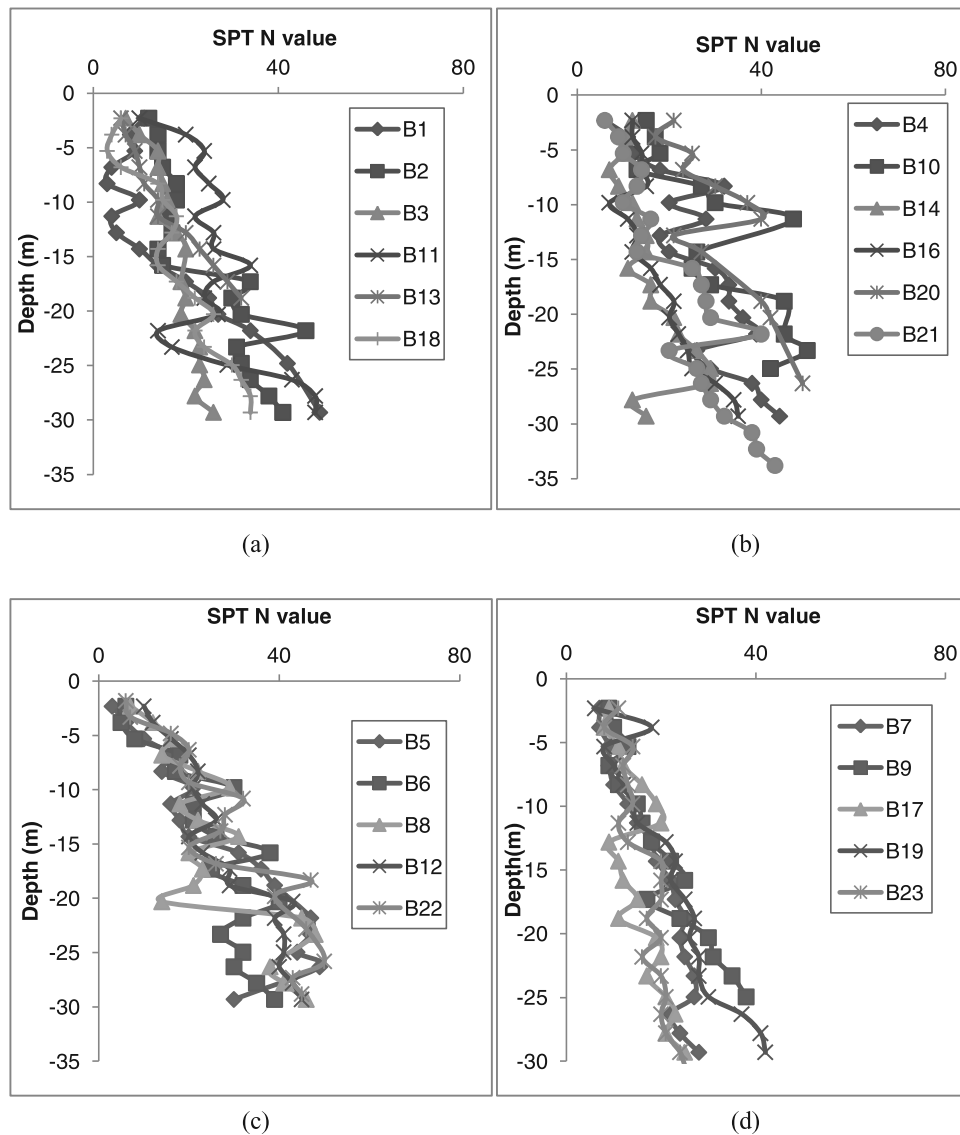


Figure 2. (a)–(d) Plot of standard penetration test (SPT) N values versus depth of data considered in the study.

form of the regression equation used for all the correlations is given below:

$$\text{Density} = a * (\text{SPT } N \text{ or } V_s)^b \quad (1)$$

where a , b are regression coefficients which vary inversely to each other.

3.1. Density and shear wave velocity

Density is an important parameter—often assumed from soil properties—for estimating the shear modulus. Here, the shear wave velocity, density and depth at 22 locations have been considered in order to develop the correlations. These 22 locations comprise 354 V_s and density data sets, which consist of 225 fine-grained soil data sets and 129 coarse-grained data sets. CL, ML, CL-ML, CI and MI were grouped as fine-grained, whereas SM, SP and SM-SP were grouped as coarse-grained soils. Statistical analysis was carried out separately for the three sets of data. Figures 5(a)–(c) depict the relation between the bulk density and V_s for all soil types as well as

fine-grained and coarse-grained soil. The relations developed, which correlate the bulk density and shear wave velocity, are given below:

$$\rho_w = 0.412V_s^{0.262} (R^2 = 0.781) \text{ for all soil types} \quad (2a)$$

$$\rho_w = 0.742V_s^{0.166} (R^2 = 0.863) \text{ for fine-grained soil} \quad (2b)$$

$$\rho_w = 0.352V_s^{0.283} (R^2 = 0.837) \text{ for coarse-grained soil} \quad (2c)$$

The form of the equation along with the constants, the standard error for 95% probability, the standard error of estimate, the correlation coefficient (r) and the coefficient of determination (R^2) have been given in table 1. Among the three relations, the relation for fine-grained data has the highest R squared value when compared to the coarse-grained data—which is more than the R squared value of all the data sets. These correlations have a good correlation coefficient and a smaller standard error of estimate. The newly developed correlations have been compared to the existing literature. Inazaki (2006) has plotted the data between the shear wave velocities and geotechnical

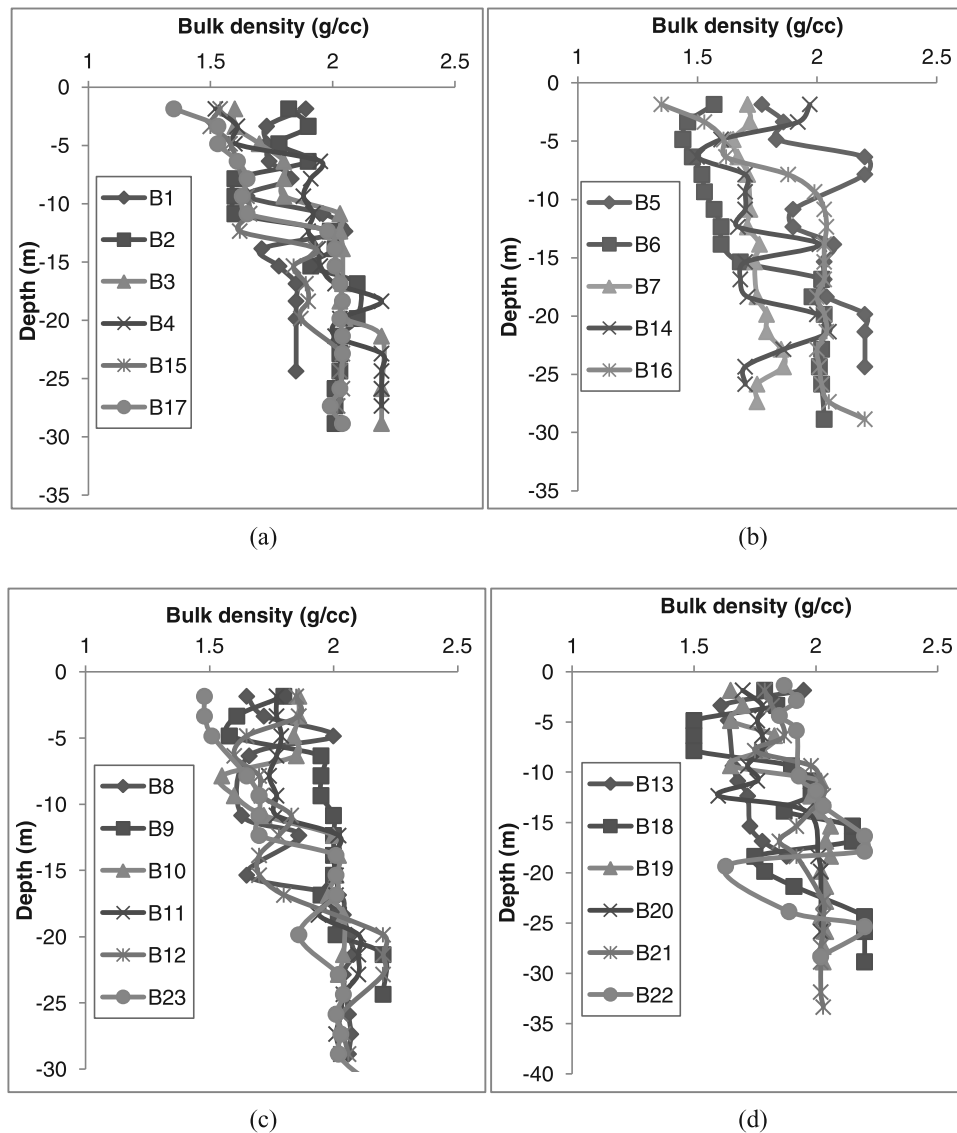


Figure 3. (a)–(d) *In situ* bulk density with depth considered in the study.

properties of alluvial sediments from all over Japan. The author obtained the shear wave velocity of surficial unconsolidated sediments from the Kanto basin and Nobi basin through PS suspension logging at seven sites and 128 boreholes. It was estimated that about half of the data was measured in Holocene sediments, which are unconsolidated, have a high water content, are soft and mainly consist of clays, silts, sands and gravels. So, the unconsolidated sediments were classified as gravel, sand, silt, or clay on the basis of mean grain size. The author defined a plot relating the shear wave velocity to the bulk density in which the Holocene sediments were divided into nine depositional units in the borehole based on detailed sedimentological analysis and the geotechnical testing of the drilling cores. Since direct correlations were not available from the mentioned paper, the data from the plot was digitized and correlations were developed for all soil types. A total of 36 points with V_s ranging from 100–450 m s^{-1} and bulk densities ranging from 1.55–1.77 g cc^{-1} for a depth of 60 m were used in deriving the relation. Figure 6 shows the data from Inazaki (2006) and the correlation developed from

all the data. The corresponding correlation is as given in equation (3a). It was observed from the paper that Inazaki (2006) plotted the data including gravel and rock, so another correlation was developed by removing the data corresponding to these materials. The relation after the removal of the gravel and rock data (the modified data of Inazaki 2006) is given by equation (3b). The modified data has higher R^2 values when compared to the data used in this study:

$$\rho_w = 0.779V_s^{0.158} (R^2 = 0.786) \text{ for all soil types} \quad (3a)$$

$$\rho_w = 0.742V_s^{0.163} (R^2 = 0.96) \text{ for all soil types} \quad (3b)$$

The proposed correlations have a higher R^2 value when compared to the Inazaki (2006) data corresponding to different soil types. A further graphical comparison is shown in figure 7, which compares the three newly developed relations (all soil types, fine-grained and coarse-grained) with the relations derived from Inazaki (2006). It can be observed from figure 7 that the relation derived from Inazaki (2006) predicts a higher

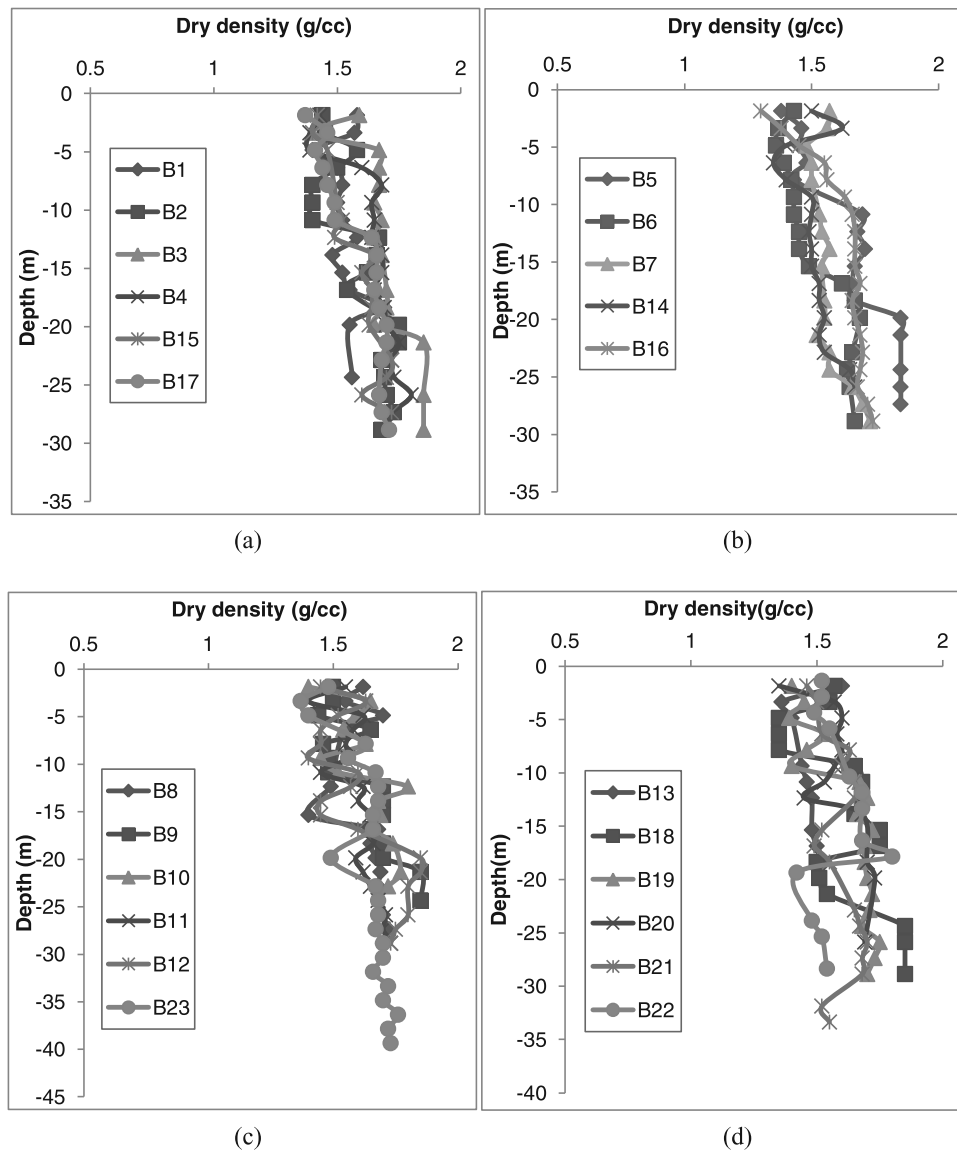


Figure 4. (a)–(d) Plot of dry density with depth considered in the study.

value than the newly developed relations for all soil types. The newly developed fine-grained relation almost coincides with the Inazaki (2006) relation. The Inazaki (2006) relation matches the newly developed correlations beyond a V_s of 250 m s^{-1} . From figure 7, it can also be noticed that the Inazaki (2006) relation has a lower R^2 value and under predicts beyond 450 m s^{-1} . Furthermore, the data in the present study is greater and has a wider range, i.e. a V_s from 100 to 650 m s^{-1} , when compared to the Inazaki (2006) data—which has less data and a maximum V_s value of 450 m s^{-1} . A comparison between the predicted and measured bulk density is given in figure 8 for all soil types. The plotted data is scattered between the lines of slope $1 : 1.2$ and $1 : 0.8$. The majority of the values lie on the line with slope $1 : 1$, showing good agreement between the measured and predicted values of the bulk density. Similar observations were noted while plotting the predicted and measured values for the fine- and coarse-grained data separately.

An attempt has also been made in this study to develop a correlation between the dry density and shear wave velocity.

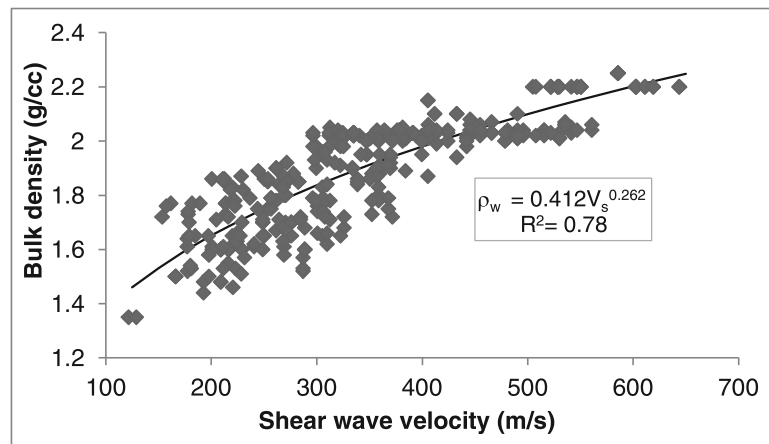
Correlations were developed using 354 data sets for all soil types, 225 data sets for fine-grained soil and 129 data sets for coarse-grained soils. The dry density values used ranged from 1.38 – 1.85 g cc^{-1} in a depth of 20 – 40 m . Figures 9(a)–(c) show the relation between the dry density and shear wave velocity for all soil types as well as fine-grained and coarse-grained soil. The newly developed correlations are:

$$\rho_d = 0.523V_s^{0.193} (R^2 = 0.80) \text{ for all soil types} \quad (4a)$$

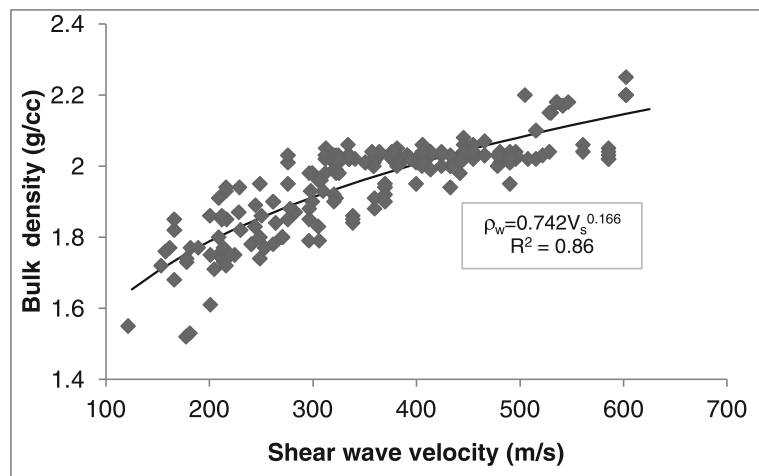
$$\rho_d = 0.981V_s^{0.090} (R^2 = 0.93) \text{ for fine-grained soils} \quad (4b)$$

$$\rho_d = 0.615V_s^{0.157} (R^2 = 0.91) \text{ for coarse-grained soils} \quad (4c)$$

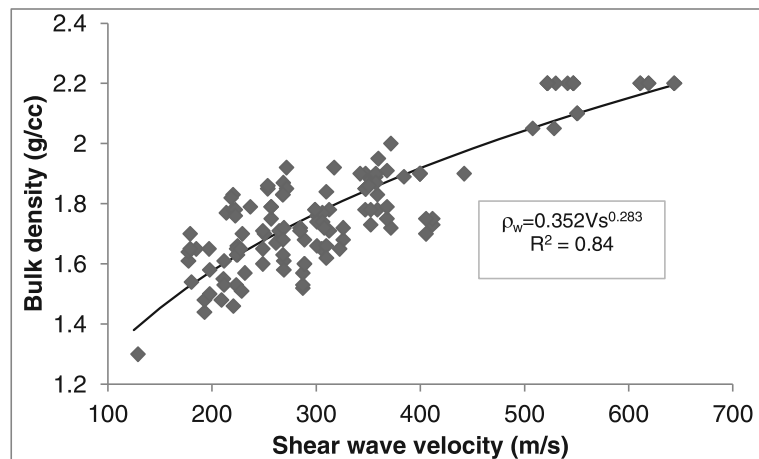
Table 1 presents the standard error for 95% probability, and the standard error of estimate, r and R^2 values. The highest R squared values were found for the fine-grained data, similar to the bulk density relations. The three relations are compared in figure 10. It can be observed that the relations for all soil types



(a)



(b)



(c)

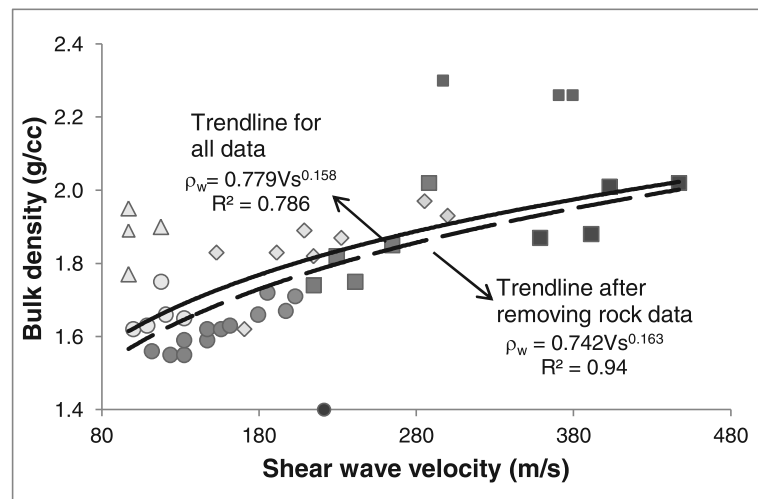
Figure 5. (a) Relation between bulk density and shear wave velocity for all soil types. (b) Relation between bulk density and shear wave velocity for fine-grained soil. (c) Relation between bulk density and shear wave velocity for coarse-grained soil.

and coarse-grained soils predict similar values of dry density at lower values of shear wave velocity. The predicted and measured dry density comparison is shown in figure 11 for all soil type data. The plotted data is scattered between the lines of

slope 1:1.2 and 1:0.8. The majority of the values lie on the line with slope 1:1, showing good agreement between the measured and predicted values of dry density. A similar observation was also noticed for fine-grained and coarse-grained soil.

Table 1. Summary of the proposed regression equations of bulk density, dry density.

Equation No. and form	Soil type	No. of data sets	Coefficients		Standard error		Standard error of estimate (s)	Correlation coefficient (r)	Coefficient of determination (R^2)
			a	b	a	b			
(1) $\rho_w = aV_s^b$	All	354	0.412	0.262	0.021	0.0087	0.103	0.884	0.781
	Fine-grained	225	0.742	0.166	0.031	0.0071	0.067	0.929	0.863
	Coarse-grained	129	0.352	0.283	0.0299	0.0146	0.104	0.915	0.837
(2) $\rho_d = aV_s^b$	All	354	0.523	0.193	0.02	0.0067	0.0675	0.894	0.800
	Fine-grained	225	0.981	0.09	0.023	0.0039	0.0323	0.966	0.930
	Coarse-grained	129	0.615	0.157	0.03	0.0085	0.05	0.953	0.910
(3) $\rho_w = aN^b$	All	364	1.232	0.141	0.0195	0.0051	0.101	0.87	0.760
	Fine-grained	229	1.67	0.059	0.0135	0.0026	0.0515	0.947	0.897
	Coarse-grained	132	1.257	0.111	0.0233	0.0062	0.0698	0.935	0.874
(4) $\rho_d = aN^b$	All	364	1.158	0.108	0.0137	0.0038	0.064	0.896	0.800
	Fine-grained	229	1.46	0.044	0.0082	0.00018	0.026	0.976	0.953
	Coarse-grained	132	1.267	0.057	0.0124	0.0033	0.0314	0.975	0.950
(5) $V_s = a\rho_w^b$	All	354	49.73	3	3.56	0.103	59.54	0.86	0.740
	Fine-grained	225	18.12	4.4	2.72	0.21	62.25	0.84	0.706
	Coarse-grained	129	61.98	2.8	4.97	0.122	54.26	0.895	0.801
(6) $N = a\rho_w^b$	All	364	0.975	4.85	0.172	0.256	7.11	0.78	0.608
	Fine-grained	229	0.038	9.2	0.0089	0.747	8.055	0.747	0.558
	Coarse-grained	132	0.96	5.47	0.224	0.387	6.32	0.811	0.658
(7) $\rho_w = aV_s^b$	All soil and rock	476	0.52	0.2	0.0144	0.0033	0.151	0.935	0.875
(8) $V_s = a\rho_w^b$	All soil and rock	476	55.88	4.06	5.688	0.105	47.1	0.935	0.875

**Figure 6.** Data from Inazaki (2006) with relation between the bulk density and shear wave velocity considering all the data (solid line) and modified data (dashed line) by removing gravel and rock data points.

3.2. Density and SPT N value

The SPT is the most widely adopted *in situ* test and is used to determine several different geotechnical properties found in subsurface soils. The SPT N value provides an indication of the relative density of the subsurface soil and is used in empirical geotechnical correlations to estimate its approximate shear strength properties. The measured SPT N values and bulk density at 23 locations were considered to develop correlations between the SPT N and *in situ* bulk density. The total obtained data was segregated into fine-grained and coarse-grained, as mentioned in the earlier sections. The whole data set comprised 361 SPT N and bulk density data points, including 229 fine-grained and 132 coarse-grained data sets.

The relation between the SPT N value and bulk density for all soil types as well as fine-grained and coarse-grained soils are shown in figures 12(a)–(c). The proposed corresponding correlations are as given below:

$$\rho_w = 1.232N^{0.141} (R^2 = 0.76) \text{ for all soil types} \quad (5a)$$

$$\rho_w = 1.67N^{0.060} (R^2 = 0.897) \text{ for fine-grained soil} \quad (5b)$$

$$\rho_w = 1.257N^{0.111} (R^2 = 0.874) \text{ for coarse-grained soil} \quad (5c)$$

The upper and lower bound equations can be obtained from the standard error values mentioned in table 1. It can be noted that the R^2 values in the above relations are relatively lower

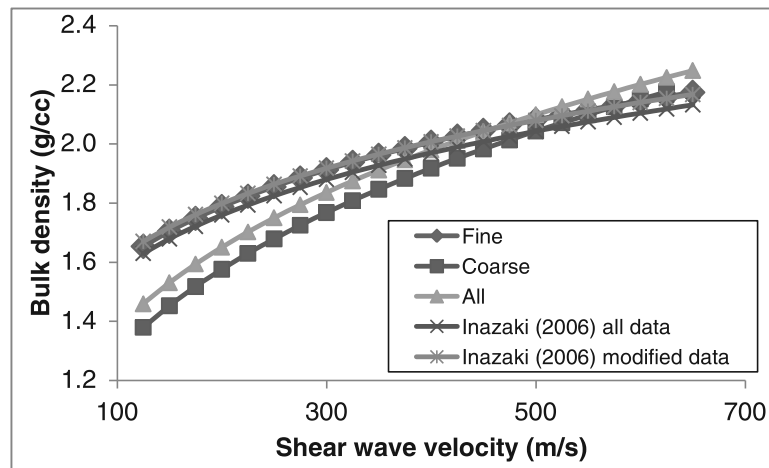


Figure 7. Comparison of bulk density versus shear wave velocity relation developed from Inazaki's (2006) data and the Inazaki (2006) modified data with the newly proposed relation in this study.

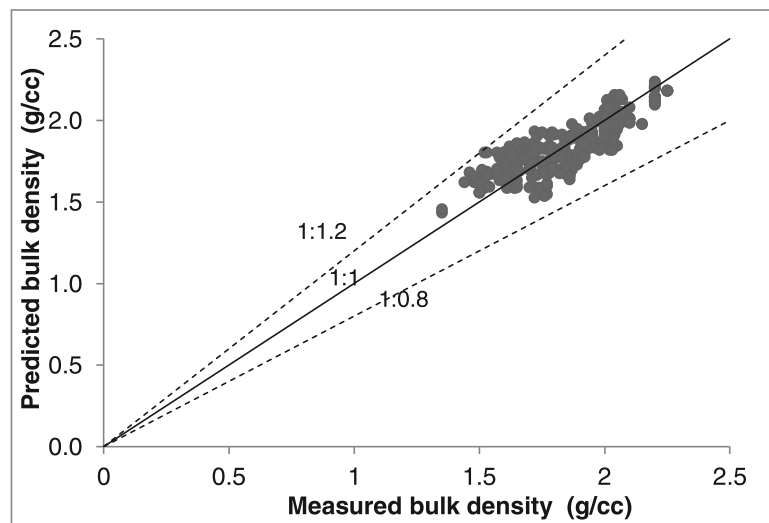


Figure 8. Plot between predicted bulk density and measured bulk density for all soil types considering shear wave velocity values.

than those of the V_s -based relation. This may be attributed to the data, the SPT N values and the fact that the bulk densities are measured in the same layer, but at different depths. Undisturbed soil samples are collected by a thin tube, and this is followed by the SPT N measurement. The approximate distance between both tubes is about 0.45 m maximum. In the case of the V_s relation, the V_s values are obtained from the profile at the respective depths of undisturbed soil sampling. A comparison of the developed correlations relating the bulk density and the SPT N value is shown in figure 13. The relation for fine-grained soils predicts a higher bulk density than those for all soil types and coarse-grained soils. All soil types and coarse-grained soil relations predict closer values up to an SPT N value of 10. The relation for all soil types coincides with that for fine-grained soils when the SPT N value is more than 40. The predicted values of the bulk density are compared with the measured values in figure 14. It was observed that the plotted values lie between the lines of slope 1:1.2 and 1:0.8. The majority of the values lie on the 1:1 line, thus implying that the predicted values are close to the measured ones. Similar results were obtained for the fine-grained and coarse-grained soils.

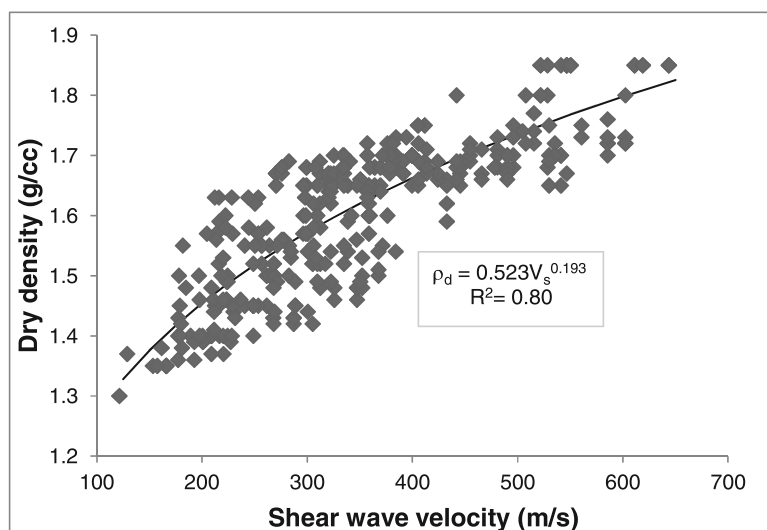
A correlation has also been developed between the dry density and SPT N value. The correlations were developed using the same number of data points discussed above. Figures 15(a)–(c) show the relation between the dry density and SPT N for all soil types as well as fine-grained and coarse-grained soil. The newly developed correlations are:

$$\rho_d = 1.158N^{0.108} \quad (R^2 = 0.80) \text{ for all soil types} \quad (6a)$$

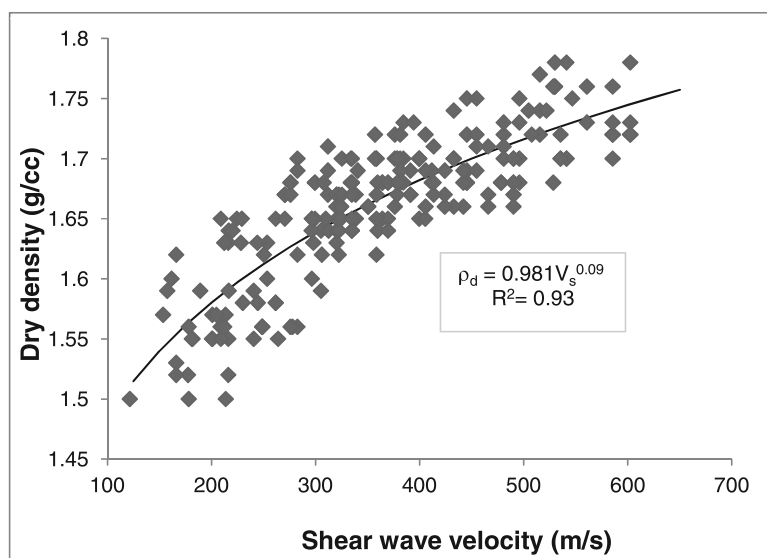
$$\rho_d = 1.46N^{0.044} \quad (R^2 = 0.953) \text{ for fine-grained soil} \quad (6b)$$

$$\rho_d = 1.267N^{0.057} \quad (R^2 = 0.95) \text{ for coarse-grained soil} \quad (6c)$$

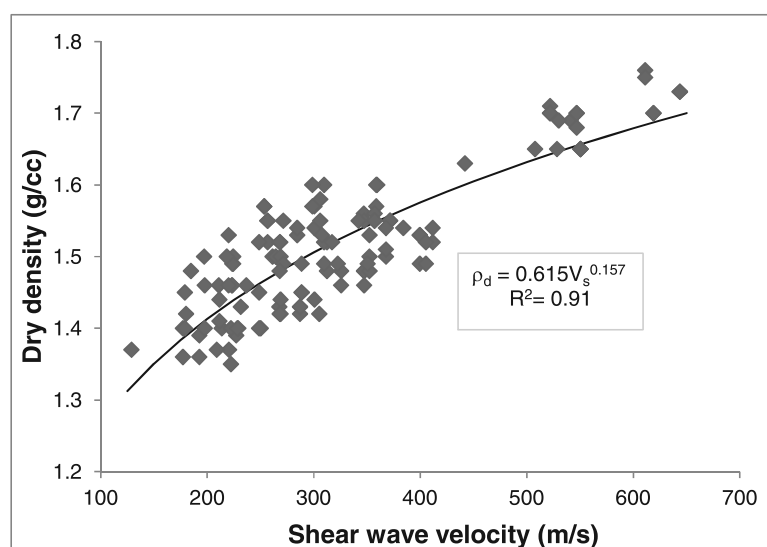
The R^2 values obtained for the SPT N and dry density correlations are more than that of the bulk density relations. A comparison of these three relations is shown in figure 16. The correlation for all soil types closely matches the coarse-grained relation up to an SPT N value of 10 and a fine-grained relation above 40. The predicted and measured dry density comparison is shown in figure 17 for all soil type data. The plotted data is scattered between the lines of slope 1:1.2 and



(a)



(b)



(c)

Figure 9. (a) Relation between dry density and shear wave velocity for all soil types. (b) Relation between dry density and shear wave velocity for fine-grained soil. (c) Relation between dry density and shear wave velocity for coarse-grained soil.

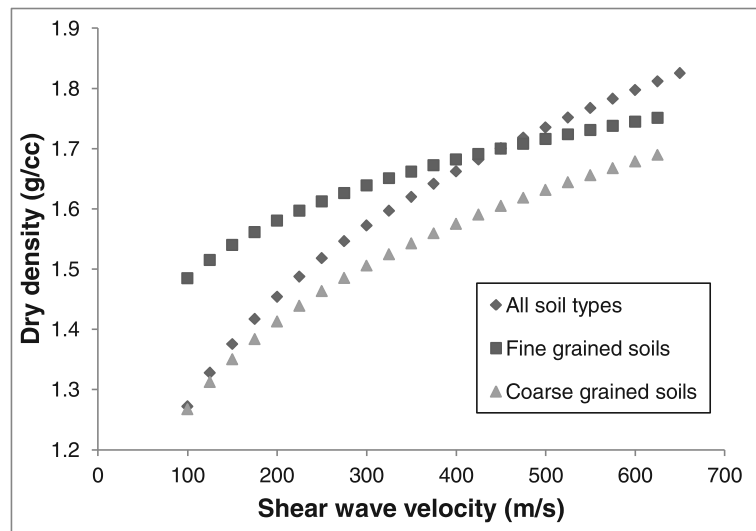


Figure 10. Comparison of dry density versus shear wave velocity relation for all soil types, fine-grained and coarse-grained soils.

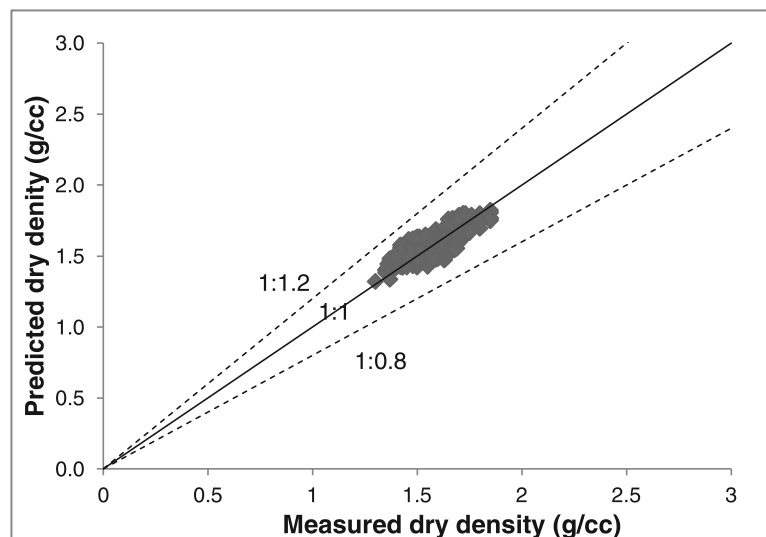


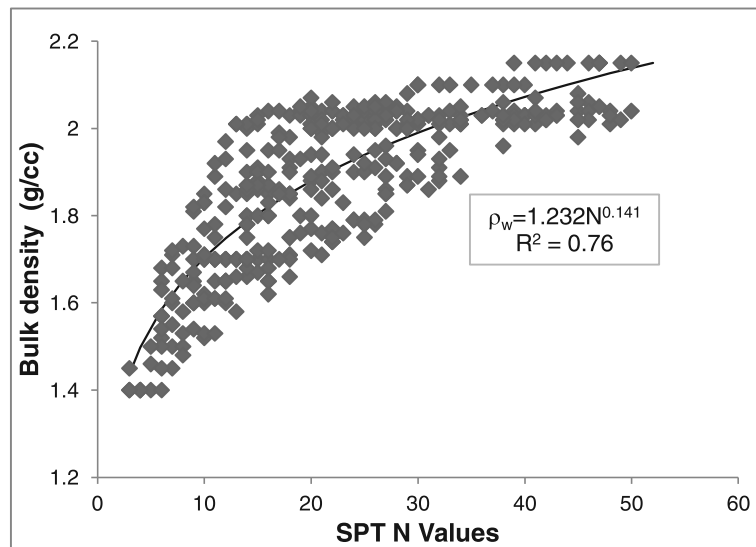
Figure 11. Plot between predicted dry density and measured dry density for all soil types considering shear wave velocity values.

1:0.8. The majority of the values lie on the line with slope 1:1, showing good agreement between the measured and predicted values of dry density. A similar observation was also noticed for fine-grained and coarse-grained soil using the developed correlations.

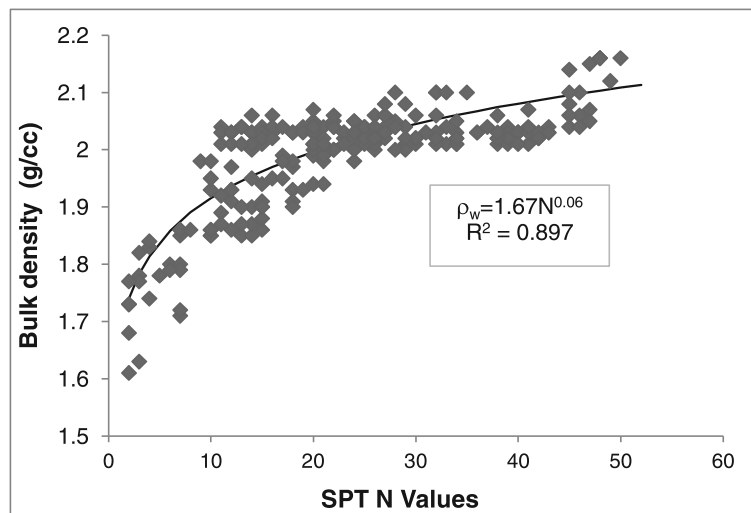
4. Results and discussions

The *in situ* density of subsurface layers is an essential geotechnical parameter for numerical modeling and analysis in many geotechnical as well as earthquake geotechnical engineering problems. Most of the time, densities are assumed based on the SPT N values or shear wave velocity. Many site response studies are currently being carried out for microzonation, and the response parameters are estimated by assuming *in situ* density values. There are several site response studies using rock- and surface- recorded earthquake data similar to the Japanese strong motion network,

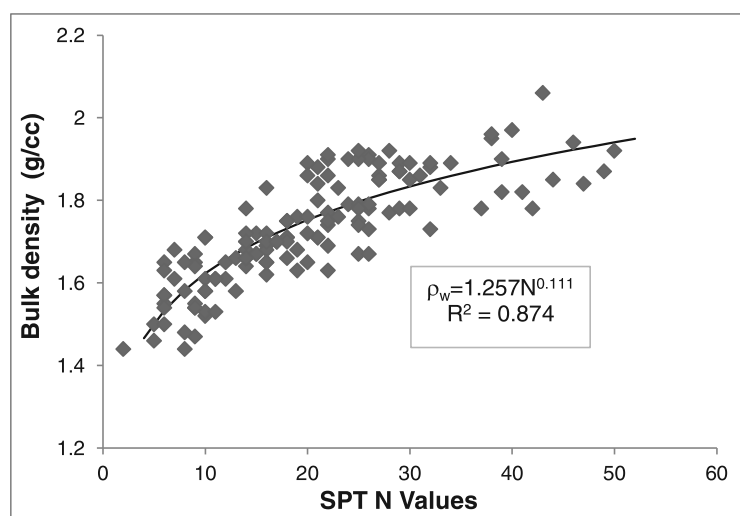
KiK-net, to validate models and existing correlations. In these databases, only the SPT N or shear wave velocity are available, and the *in situ* densities are either assumed or calculated by using the existing density relations for rock (Kaklamanos *et al* 2013). The most widely used density relation in site response studies was developed by Gardner *et al* (1974) and considers rock data. Boore (2007) suggested the procedure for estimating the *in situ* density of soils using relations from Brocher (2005) and Gardner *et al* (1974). As per the procedure of Boore (2007), a density of 1.8 g cc^{-1} can be obtained for a shear wave velocity of 0.1 km s^{-1} . A review of the literature shows that very limited direct correlations are available for the prediction of *in situ* density. In this work, 23 boreholes with SPT N values and 22 shear wave velocity profiles were used to generate a correlation between the index properties of the bulk and dry density and the engineering properties of the SPT N and shear wave velocity. Table 1 presents a summary of all the developed correlations along with their respective statistical



(a)



(b)



(c)

Figure 12. (a) Relation between bulk density and SPT N value for all soil types. (b) Relation between bulk density and SPT N value for fine-grained soils. (c) Relation between bulk density and SPT N value for coarse-grained soils.

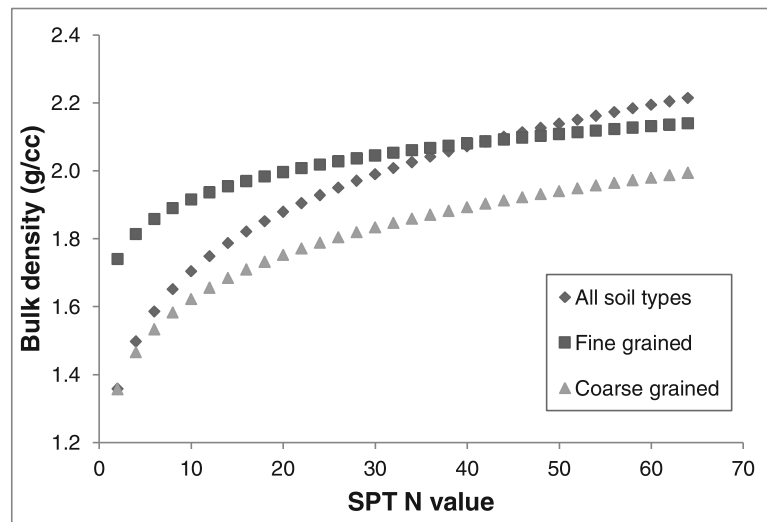


Figure 13. Comparison of the proposed correlations between bulk density and SPT N value for all soil types as well as fine-grained and coarse-grained soils.

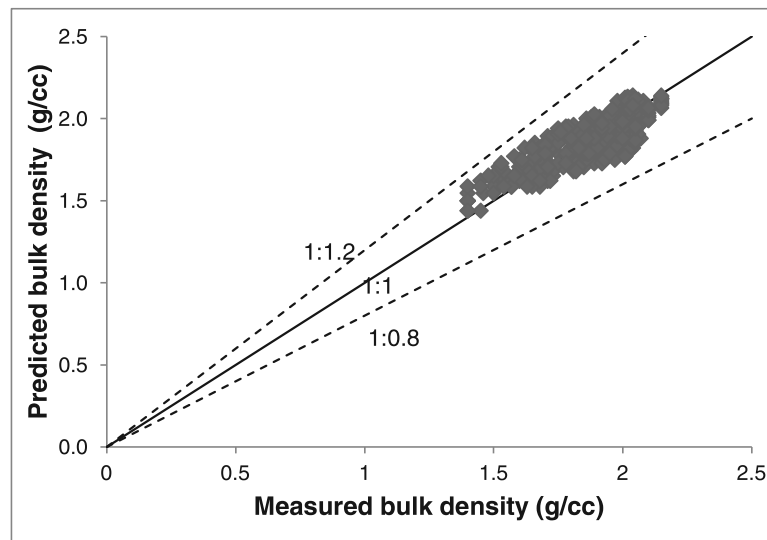


Figure 14. Plot between predicted bulk density and measured bulk density for all soil types considering SPT N values.

parameters, proving their credibility. Apart from giving a direct correlation between the index and engineering properties, a reverse correlation between the SPT N and V_s with *in situ* density has been developed, as shown in the last two rows of table 1. The correlation coefficients and R^2 values for the reverse correlations are relatively lower than the direct correlation, even though the data used was the same. A reliability and performance analysis has been established for all direct correlations, as given in table 1, based on the normalized consistency ratio, C_d (Dikmen 2009), determined as:

$$C_d = \frac{\rho_M - \rho_C}{V_s} \quad (7)$$

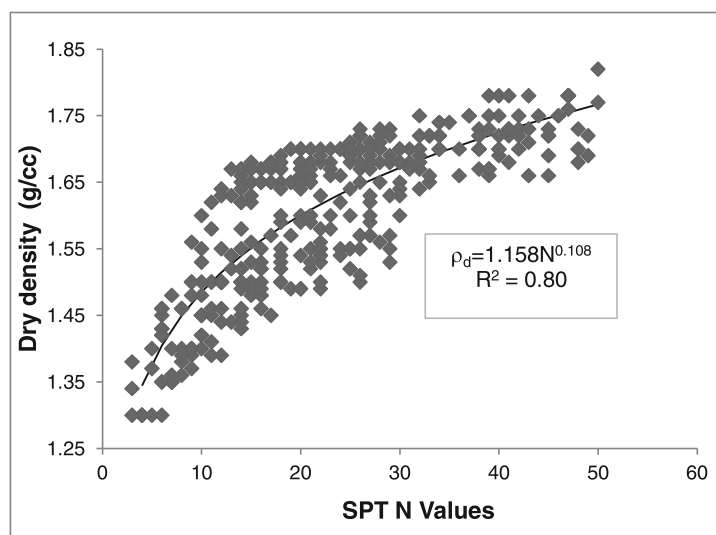
where ρ_M and ρ_C are the bulk density values measured and calculated from the correlations developed. A typical plot of the consistency ratio for the bulk density versus the shear wave velocity is given in figure 18. Figure 18 shows that the C_d values are close to zero for all shear wave velocities. Normalized consistency ratio curves were plotted for all the

developed equations and it was found that the values of C_d lie close to zero, which means that all the proposed equations have good prediction performance.

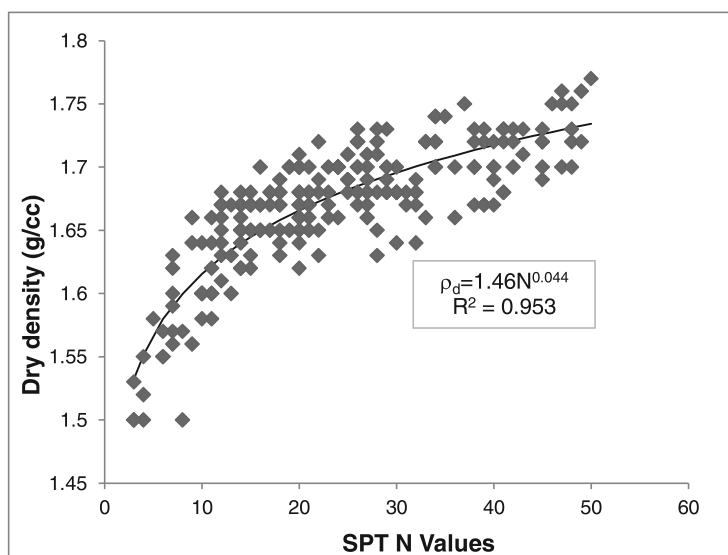
To estimate the capability of the new proposed relationship, a scaled percentage error, E_r (Dikmen 2009), was calculated by using the equation given below:

$$E_r = 100(\rho_C - \rho_M)/\rho_C \quad (8)$$

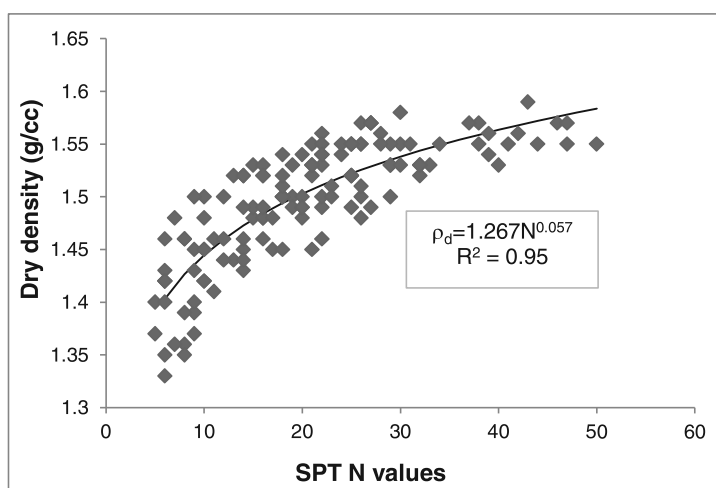
where ρ_C is the calculated or predicted value of the bulk density and ρ_M is the measured value of the bulk density. The graph between the scaled error and cumulative frequency for the relations developed for the bulk density of all soil types as well as fine-grained and coarse-grained soils is shown in figure 19. From figure 19, it is clear that using the relation for all soil types, about 99% of the bulk density values are within an 18% error margin. Using the equation for fine-grained soil, it was observed that 95% of the bulk density values were predicted within a margin of 15% error. For coarse-grained soils, almost 94% of the bulk density values were predicted within



(a)



(b)



(c)

Figure 15. (a) Relation between dry density and SPT N value for all soil types. (b) Relation between dry density and SPT N value for fine-grained soil. (c) Relation between dry density and SPT N value for coarse-grained soil.

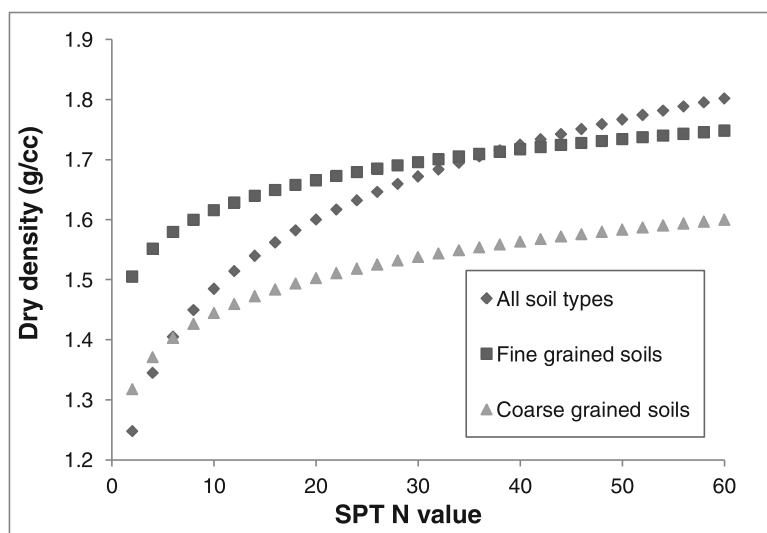


Figure 16. Comparison of the proposed correlations between the dry density and SPT N value for all soil types as well as fine-grained and coarse-grained soils.

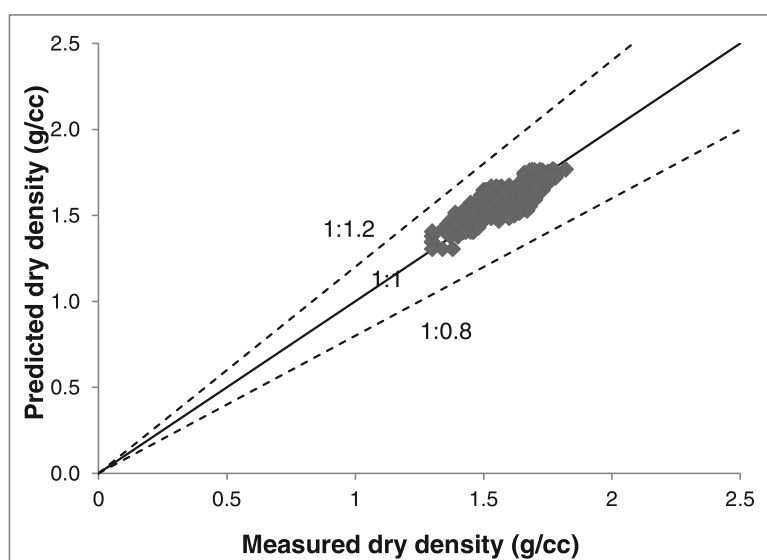


Figure 17. Plot between predicted dry density and measured dry density for all soil types considering SPT N values.

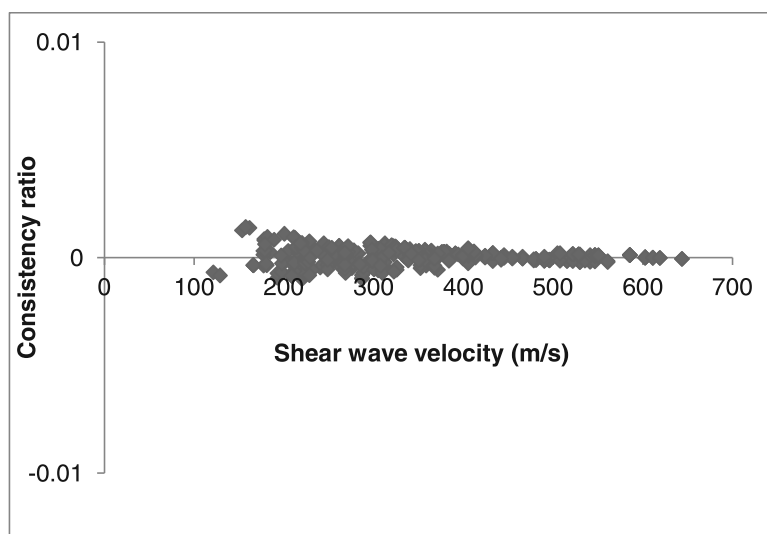


Figure 18. Typical plot of the normalized consistency ratio of bulk density and V_s for all soil types.

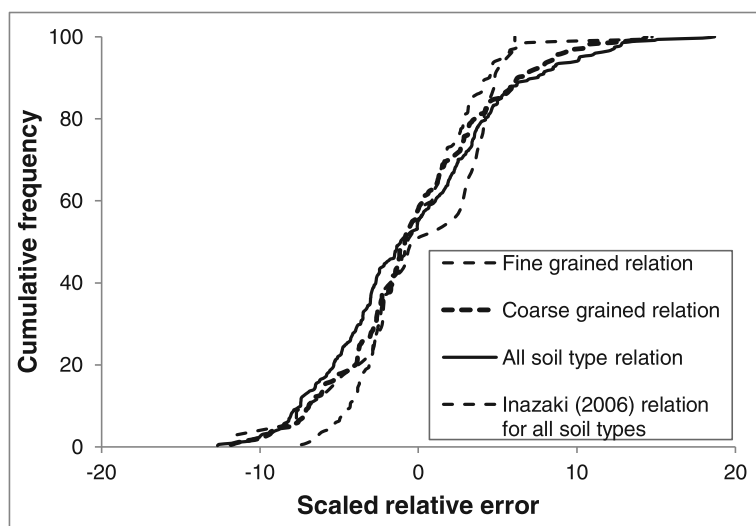


Figure 19. Scaled relative errors of bulk density predicted from V_s for all soil types as well as fine-grained and coarse-grained soils.

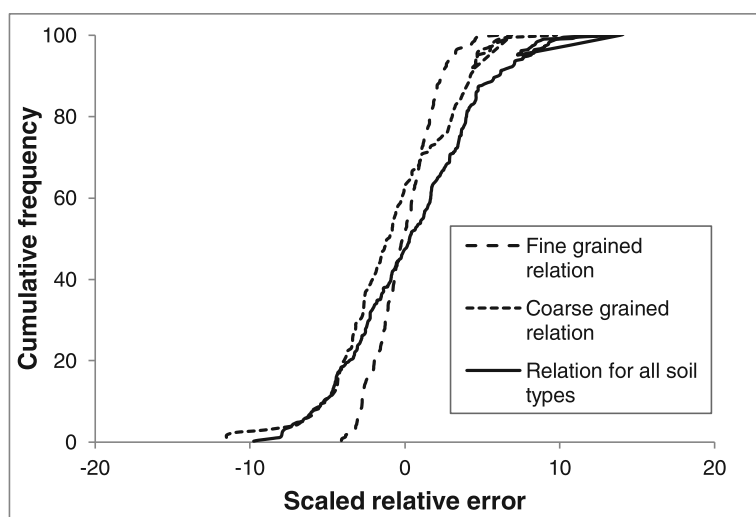


Figure 20. Scaled relative errors of dry density predicted from V_s for all soil types as well as fine-grained and coarse-grained soils.

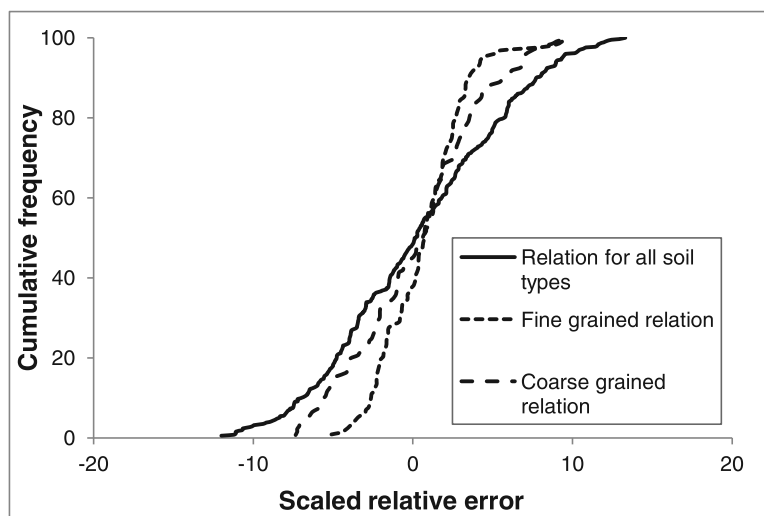


Figure 21. Scaled relative errors of bulk density predicted from SPT N for all soil types, as well as fine-grained and coarse-grained soils.

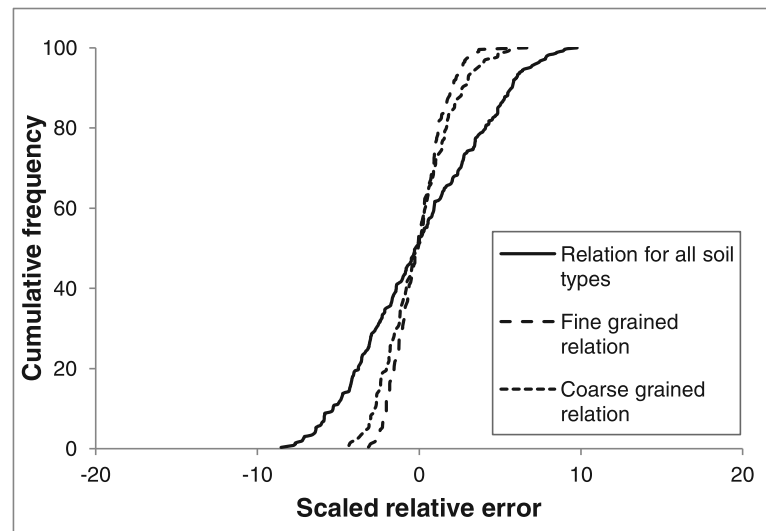


Figure 22. Scaled relative errors of dry density predicted from SPT N for all soil types, as well as fine-grained and coarse-grained soils.

a 10% error margin. These results show that the proposed relationships for all soils as well as fine-grained and coarse-grained soils provide a good estimation of the bulk density. Figures 19–22 show the scaled error and cumulative frequency for the relations developed in this study. It can be observed that the scaled errors of all the relations vary between $\pm 20\%$, indicating that they are all able to predict values close to the measured values with less error. These relations can be used to estimate densities using the V_s and SPT N values for geotechnical engineering applications.

It can be noted from the study that the data and correlations are suitable for soil layers, but that the density of the rock layers is also needed during the process of site response studies. In order to generate the correlation between the bulk density and shear wave velocity for soil and rock, the data from previous studies was digitized. Brocher (2005) and other researchers published the data on the bulk density and compressional/shear wave velocity of dry and fully saturated rocks (Boore 2007). Brocher (2005) highlighted that this data was derived from igneous, sedimentary and metamorphic rock samples (Boore 2007). The compiled data for rock, presented by Boore (2007), was digitized and about 122 sets of bulk density and compressional wave velocity data were obtained. These compressional wave velocities were converted into shear wave velocity using Brocher's regression fit (Brocher 2005) relation between V_s and V_p . The shear wave velocity and bulk density values for the rock data varied from between $130\text{--}4000\text{ m s}^{-1}$ and $1.26\text{--}3.05\text{ g cc}^{-1}$. This data was added to the densities and shear wave velocities from the study area, and a new correlation was developed, as given in equation (9). An inverse relation was also determined to obtain the shear wave velocity from the values of the bulk density, as shown in equation (10). The trend between the new correlation and the inverse relation given by the following equations can be seen in figures 23(a) and (b).

$$\rho_w = 0.52V_s^{0.2} \quad (R^2 = 0.875) \quad (9)$$

$$V_s = 55.88\rho_w^{4.06} \quad (R^2 = 0.875) \quad (10)$$

It can be seen from the above equations and from table 1 that all the data has a high R^2 value, implying good prediction capability similar to the equations proposed in the earlier sections. The predicted values were compared with the measured values of the bulk density, and these values were scattered between the lines of slope 1 : 1.5 and 1 : 0.8, with almost all values lying on the line with slope 1 : 1. The cumulative frequency graphs were plotted against the scaled error and it was observed that 97% of the values were within an error margin of 20%. A comparison was made between the proposed relations for all soil and rock, and Boore (2007) suggested the Gardner relation in figure 24. From figure 24, it can be observed that the relation suggested by Boore (2007) predicts higher values of density than the proposed relations. It can also be noted that there is a difference in the predictions of the proposed relation and the Gardner relation suggested by Boore (2007) up to 1000 m s^{-1} , which might be due to the extrapolation of the curve between the V_s values from $100\text{--}300\text{ m s}^{-1}$. The relations developed by Boore (2007) are valid for the shear wave velocity values ranging from $300\text{--}3550\text{ m s}^{-1}$, whereas the proposed relations are valid from $100\text{--}4000\text{ m s}^{-1}$. Therefore, it can be concluded that the proposed relation may be utilized for all soil and rock effectively with adequate accuracy. The proposed correlations—considering the shear wave velocity can be used in any region as the V_s measurement by seismic surface wave methods—may not differ significantly from region to region; however, correlations considering the SPT N values cannot be used in other regions directly. The SPT N values depend on the hammer energy and other parameters, which are region specific and may vary from region to region. The SPT N values depend on drilling methods, drill rods, borehole sizes and stabilization, the sampler, blow count rate, hammer configuration, energy corrections, fine content and test procedure (Schmertmann and Palacios 1979, Kovacs *et al* 1983, Farrar *et al* 1998, Sivrikaya and Togrol 2006, Anbazhagan *et al* 2012). These correlations can be used for other regions if proper correction factors are applied to normalize the SPT N values. The first and foremost correction factor is that relating to the hammer energy, which depends on the energy applied

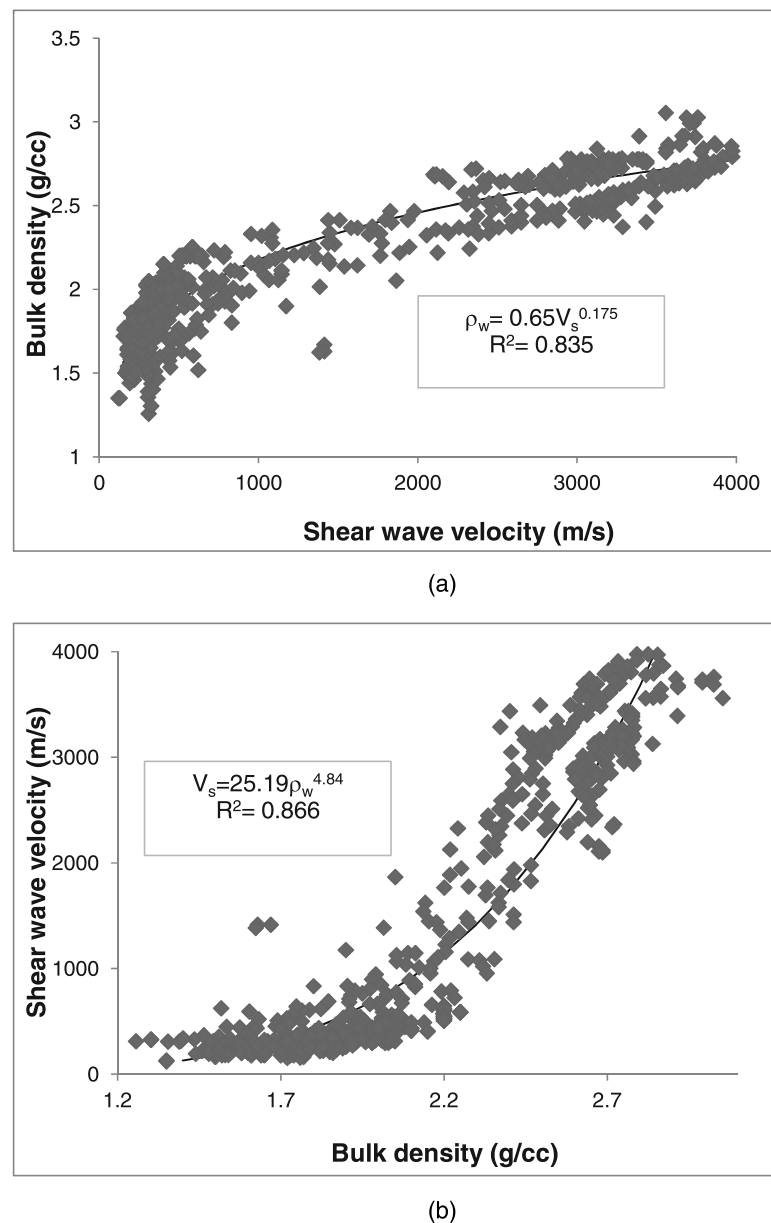


Figure 23. (a) Relation to obtain the bulk density from the shear wave velocity of all soil and rock (after considering the data from the present study and from Boore 2007). (b) Relation to obtain the shear wave velocity from the bulk density of all soil and rock (after considering the data from the present study and from Boore 2007).

to count the N values. The SPT N values are measured in India as per IS:2131 (1981), which is similar to the ASTM D6066-96 procedure. Even though the hammer energy measurement is mandatory for estimating the normalized SPT N values according to ASTM D1586-99, hammer energy is not currently measured during the SPT in India. Recently, an attempt was made by the first author and team to measure the hammer energy below the anvil and above the split spoon sampler by building a new indigenous SPT-hammer energy measurement apparatus (SPT-HEMA). The SPT-HEMA is capable of measuring force and velocity signals at different depths as well as the energy transferred to the drill rods and sampler. Preliminary field studies in selected sites around India showed that the typical energy ratio below the anvil is about 60% (Selvam *et al* 2013). It was inferred from this study

that only 60% theoretical energy was transferred to the SPT rod in India. Regions with a similar energy transfer ratio can use the proposed correlations directly and regions with different energy transfer ratios need to apply the necessary correction factor, which is mentioned in Anbazhagan *et al* (2012). Furthermore, in this study the same set of velocity–density at different depths was used for the correlation. Figure 25 shows a plot of the selected set of velocity and density with depth. It can be noted here that at a particular depth, the data set of V_s and density is insufficient to generate a depth-based correlation. Moreover, the data is only available for soil and up to a depth of 40 m. Large sets of V_s and density data may be required for the same depth in a wide range of soil and rock to develop depth-dependent correlations, and may be complied in the future to generate a depth-dependent V_s and density

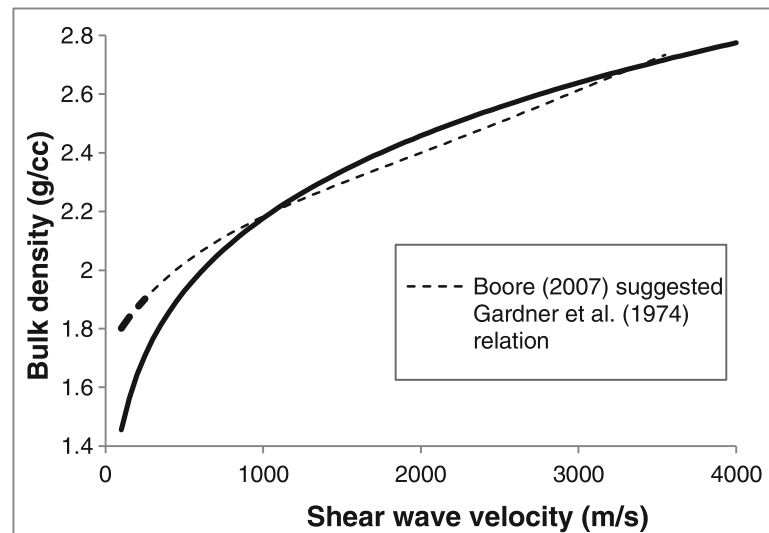


Figure 24. Comparison of Boore (2007) suggested Gardner *et al* (1974) relation and the proposed relation between the shear wave velocity and bulk density for all soil and rock. The thick dotted line shows the extrapolated region of the Boore (2007) relation up to a V_s of 300 m s^{-1} , the dashed line represents the same relation from a V_s of $300\text{--}3550 \text{ m s}^{-1}$.

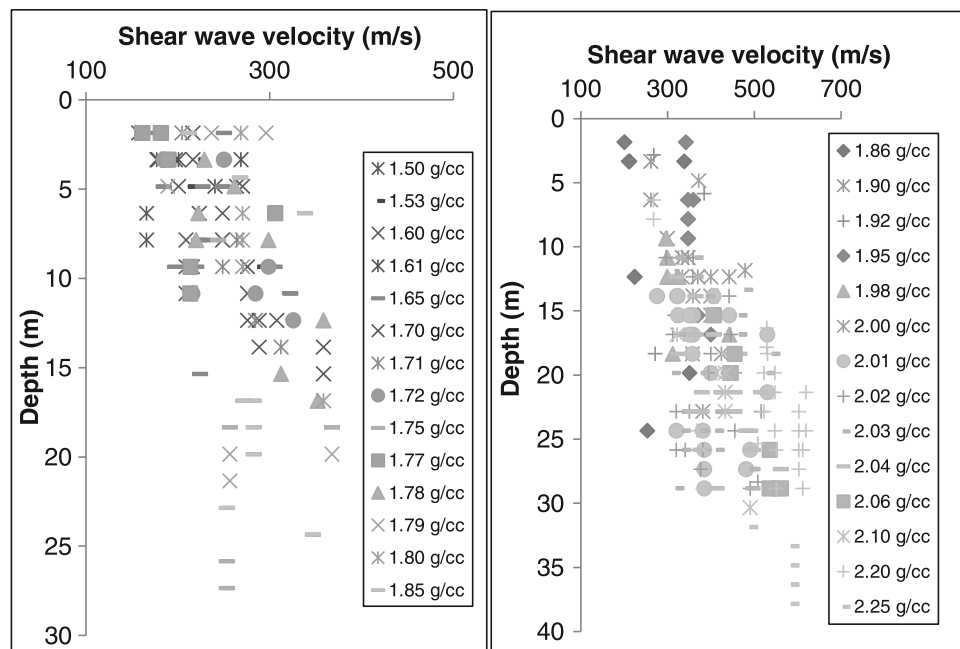


Figure 25. Variation of the density and shear wave velocity with depth for soil up to a depth of 40 m.

correlation. The proposed correlations in this study may not be directly applicable in the case of depth-dependent density and velocity estimations.

5. Conclusions

Measurements of the SPT N value, the densities (bulk and dry) and shear wave velocities obtained at 23 drilled boreholes in 22 locations using the MASW were collected and analyzed in this study. In general, density is found to increase with depth in most of the boreholes; however, in some locations, a decrease in density was also observed. This might have been due to geological parameters which were not investigated. The

results obtained from the SPT, MASW, undisturbed samples and drilled boreholes were used in developing new correlations between the density (both bulk and dry) and measured values of the shear wave velocity and SPT N value. Different correlations were developed for all soil types, as well as fine-grained and coarse-grained soils. About 95–100% of the predicted values were found to be within an error margin of 20%, thus indicating good prediction capability. The relations between the bulk density and shear wave velocity were compared with the relations derived from Inazaki (2006), and it was found that the proposed relations predicted values close to the measured values applicable to a wide range of density and V_s values. Furthermore, a graphical comparison and statistical validation was carried out. The majority of the correlations were developed for soil

layers—but it is also necessary to determine the density of rock layers as part of site response studies. Therefore, a correlation was developed between the bulk density and shear wave velocity for all soil and rock by adding the data from Boore (2007) to the data from the study area. An inverse correlation was also developed using the data mentioned above. A comparison has been made between the proposed relations for all soil and rock using the Gardner relation suggested by Boore (2007), and it was found that the proposed relations gave a good estimate of densities. The relations proposed by Gardner *et al* (1974) are only applicable for materials with a velocity of more than 1524 m s⁻¹, and not applicable for unsaturated sediments near the surface. However, the proposed relation in this study (equation (9)) can be used for all materials (from loose soil to very hard rock) with shear wave velocities of 100 m s⁻¹ to 4000 m s⁻¹. The prediction of density from field-recorded parameters like shear wave velocity and the standard penetration number can be very useful in the evaluation of the shear modulus. The relations developed are based on soil data measured *in situ* and can help in reducing errors associated with the assumption of density in geotechnical engineering problems. The proposed correlations in this study can be useful for site response and microzonation studies. It should also be noted that these empirical correlations depend on the data used in the process, and that the data should be calibrated for use in other soil conditions. These correlations can be used as a guide to complement and verify information relating to the project.

Acknowledgments

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