TECHNICAL NOTE

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Relationship between Low Strain Shear Modulus and Standard Penetration Test N Values

ABSTRACT: A low strain shear modulus plays a fundamental role in the estimation of site response parameters. In this study an attempt has been made to develop the relationships between standard penetration test (SPT) N values and the low strain shear modulus ($G_{\text{max}}$). For this purpose, field experiments SPT and multichannel analysis of surface wave data from 38 locations in Bangalore, India, have been used, which were also used for seismic microzonation project. The in situ density of soil layer was evaluated using undisturbed soil samples from the boreholes. Shear wave velocity ($V_s$) profiles with depth were obtained for the same locations or close to the boreholes. The values for low strain shear modulus have been calculated using measured $V_s$ and soil density. About 215 pairs of SPT N and $G_{\text{max}}$ values are used for regression analysis. The differences between fitted regression relations using measured and corrected values were analyzed. It is found that an uncorrected value of N and modulus gives the best fit with a high regression coefficient when compared to corrected N and corrected modulus values. This study shows better correlation between measured values of N and $G_{\text{max}}$ when compared to overburden stress corrected values of N and $G_{\text{max}}$.

KEYWORDS: shear modulus, SPT, MASW, correlation

Introduction

Site amplification of seismic energy due to local soil conditions causing damage to built environment was amply demonstrated by many earthquakes during the past century (Guerrero earthquake (1985) in Mexico City, Spitak earthquake (1988) in Leninakan, Loma Prieta earthquake (1989) in San Francisco Bay Area, Kobe earthquake (1995) in Japan, Kocaeli earthquake (1999) in Turkey, and Bhuj earthquake (2001) in India). The recent 2001 Bhuj earthquake in India is another example, with notable damage at a distance of 250 km from the epicenter (Sitharam et al. 2001; GovindaRaju et al. 2004). These failures resulted due to effects of local soil conditions on the ground motion. Many cases have shown the changes in amplitude, period, and frequency of rock motion due to soil condition. Local site conditions also modify the spectral content and duration of ground motions. The response of a soil deposit depends on the frequency of the base motion and the geometry and dynamic properties of the soil layer above the bedrock. The seismic microzonation requires the shear wave velocity and shear modulus as input to estimate site specific ground response parameters (Anbazhagan and Sitharam, 2008a and 2009b; Sitharam and Anbazhagan, 2008a; Anbazhagan et al. 2009). The site specific ground response study requires the soil parameters of thickness ($h$), density ($\rho$), and shear modulus ($G_{\text{max}}$) of each layer as an input. The soil type and thickness of each layer are generally obtained by drilling boreholes and logging the borehole information (borelog). The in situ densities of each layer are usually obtained from the undisturbed soil samples collected in boreholes. In most cases, the shear modulus ($G_{\text{max}}$) for site response analysis is evaluated using relationships based on the standard penetration test (SPT) N values. These relationships are region specific, which depends on the type and characteristics of the soil in the respective region. It is not always fair to use existing correlations to obtain shear modulus for ground response studies without considering local soil condition. Hence, in this paper an attempt has been made to develop a relationship between SPT N value and $G_{\text{max}}$ considering SPT and multichannel analysis of surface wave (MASW) data from Bangalore, India, where residual soils are abundant.

MASW is a seismic refraction method, which is widely used for sub-surface characterization. MASW is increasingly being applied to earthquake geotechnical engineering problems of seismic microzonation and site response studies (Anbazhagan and Sitharam 2008b). MASW can also be used for the geotechnical characterization of near surface materials (Park et al. 1999; Xia et al. 1999; Miller et al. 1999; Kanli et al. 2006; Anbazhagan and Sitharam 2008c). In particular, it is used in geotechnical engineering to measure the shear wave velocity and dynamic properties (Sitharam and Anbazhagan 2008b) and to identify the sub-surface material boundaries and spatial variations of rocks (Anbazhagan and Sitharam 2009a). MASW is also used in the railway engineering to identify the degree of fouling and type of fouling (Anbazhagan et al. 2010). In this paper the low strain shear modulus ($G_{\text{max}}$) is evaluated using measured shear wave velocity obtained from MASW system and in situ density from undisturbed soil samples obtained at the same depth in the corresponding boreholes. These values are used to generate a correlation between SPT measured and corrected “N” and $G_{\text{max}}$. The developed relationships are compared with a similar relationship available in the literature.

General Setting of the Study Area

The area of study is limited to the Bangalore Metropolitan Area (Bangalore Mahanagar Palike) of about 220 km². Bangalore is
situated on latitude 12°58’ North and longitude 77°36’ East and is at an average altitude of around 910 m above mean sea level. The basic geomorphology of the city comprises of a central Denudational Plateau and Pediment (towards the West) with flat valleys that are formed by the present drainage patterns. The central Denudational Plateau is almost void of any topology, and the erosion and transportation of sediments carried by the drainage network gives rise to the lateritic clayey alluvium seen throughout the central area of the city. The soil is mainly a product of strong weathering, ferruginous, clay mixture, and well drained. The main types of soil found here are red alluvium, sandy silts, alluvial clay, weathered rock (gravels), and soil fill material. The soil fill materials are a mixture of loose soil (excavated from constructions sites) and stones or building construction waste. Red alluvium (laterite) tropical residual soil is formed due to the erosion of the granitic and gneissic base rocks; this alluvium is ferruginous and is generally encountered in a clayey matrix. The erosion is caused by the natural drainage grid of lakes and streams throughout the city. Weathered rocks are generally granitic in composition; they are weathered from the parent rock and eventually combine with the sandy/clayey matrix.

Testing Programme

The field test locations for MASW were selected based on three criteria: (1) Sampling the range of soil types and conditions based on the SPT data, (2) flat surface free from noise, and (3) important places. The field MASW surveyed locations with SPT boreholes in Bangalore are shown in Fig. 1. The test locations were selected in such a way that these represent the entire city sub-surface information. In total, 38 one-dimensional (1D) MASW surveys have been carried out close to SPT borehole.

Multichannel Analysis of Surface Wave Testing

MASW is a geophysical method that generates a shear wave velocity ($V_s$) profile (i.e., $V_s$ versus depth) by analyzing Raleigh-type surface waves recorded on a multichannel. A MASW system consisting of 24 channels Geode seismograph with 24 vertical geophones of 4.5 Hz capacity has been used in this investigation. The seismic waves are created by impulsive source of 15 lb (sledge hammer) with 300 $\times$ 300 mm$^2$ size hammer plate with number of shots. Figure 2 shows a typical MASW setup in the field with source and geophone arrangements. The source created waves are captured by receivers and further used for dispersion and inversion analysis. The optimum field parameters of source to first and last receiver, receiver spacing, and spread length of survey lines are selected in such a way that the required depth of information can be obtained. These field parameters are in conformity with the recommendations of Park et al. (2002).

Multichannel Analysis of Surface Wave Data Processing

The captured seismic waves through geophones are recorded for time duration of 1000 ms. The quality of the recorded data is verified in the field itself. Noisy records are rerecorded to get better signals of record. Typical recorded surface wave arrivals for a source to first receiver distance of 5 m and the processed data are shown in Fig. 3. Figure 3(a) and 3(b) shows the data before and after controlling noise during recording time, and Fig. 3(c) shows the noise reduced data (removing the noise during data processing). These recorded data are further used to get dispersion curves, which are used to extract shear wave velocity at the mid point of testing locations. Phase velocity can be calculated from the linear slope of each component on the swept-frequency record. The lowest analyzable frequency in this dispersion curve is around 4 Hz,
and the highest frequency is 75 Hz. A typical dispersion curve along with signal amplitude and signal to noise ratio is shown in Fig. 4. Each dispersion curve is generated for corresponding signal to noise ratio of about 80 and above.

A shear wave velocity profile has been calculated using an iterative inversion process that requires the dispersion curve developed earlier as input. A least-squares approach allows automation of the process (Xia et al. 1999), which is inbuilt in SurfSeis. Typical 1D \( V_s \) profile obtained using MASW is shown in Fig. 5. The shear wave velocity values obtained from each survey line for the different layers fall within the recommendations of National Earthquake Hazards Reduction Program "\( V_{s} \)-soil classification for different site categories IBC (2006) classification. Shear wave velocity obtained for the study area is used to estimate equivalent soil overburden \( V_{so} \) and is presented in Fig. 6. The average shear wave velocity for the depth \( d \) of soil is referred as \( V_{H} \). The equivalent shear wave velocity up to a depth \( H \) \( (V_{H}) \) is computed as follows:
where:

\[ H = \sum d_i \]  

\[ d_i \text{ and } v_i \text{ denote the thickness (in m) and shear wave velocity (at a shear strain level of } 10^{-5} \text{ or less, m/s) of the } i\text{th formation or layer respectively, in a total of } n \text{ layers within the depth } H. \]

The equivalent soil shear wave velocity varies from 100 to 450 m/s. Most of the area, having an average shear wave velocity of 200–300 m/s, can be classified as medium to dense soil.

**Geotechnical Data**

The SPT is one of the oldest, most popular, and most common in situ tests used for soil exploration in soil mechanics and foundation engineering. This test is being used for many geotechnical projects because of the simplicity of the equipment and test procedure. In particular SPT tests are used for seismic site characterization, site response, and liquefaction studies towards seismic microzonation. This test is quite crude and depends on many factors due to the variations of applications carried out in the test and some equipment used in the test. The many factors includes the drilling methods, drill rods, borehole sizes and stabilization, sampler, blow count rate, hammer configuration, energy corrections, fine content, and test procedure (Schmertmann and Palacios 1979; Kovacs et al. 1981; Farrar et al. 1998; Sivrikaya and Toğrul 2006). The combined effect of all these factors can be accounted by applying the correction factors separately or together. The SPT \( N \) values may vary even for identical soil conditions because of sensitivity to operator techniques, equipment, malfunctions, and poor boring practice. So the SPT based correlations may be used for projects in preliminary stage or where there is a financial limitation, but for important projects, it is preferable to measure dynamic properties directly by using suitable field tests (Anbazhagan and Sitharam 2008c).
Boreholes used in this study have a diameter of 150 mm, drilled using hydraulic rotary drilling rigs up to the hard stratum. SPT tests were conducted at a regular sampling interval of 1.5 m in each borehole, and additional disturbed soil samples were also collected. Most of the penetration resistances (SPT N values) in the boreholes were measured using donut hammer. The undisturbed soil samples are collected according to Indian Standard IS 2132 (1986). The undisturbed soil sample is procured by driving the thin walled sampler of diameter 100 and 450 mm length tube into the borehole at desired depth/ change of strata. To avoid densification of soil samples due to hammering, the following precautionary steps are followed in the field: (1) The length of tube is marked on the driving rod and the driving is carried out carefully up to tube length (up to mark) by adjusting hammer height of fall, and (2) usually, the height of the hammer fall is limited to 20–40 cm. After ensuring complete penetration due to hammering, the tube is turned at least for two revolutions to shear the sample off at the bottom. The loose/disturbed soil in the upper end is removed on either end and taken to the laboratory. These samples are used to evaluate in situ densities of the soil layers. In most of the locations, the boreholes are drilled up to weathered rock and at few locations boreholes reached up to hard rock. A typical borehole with SPT N values with depth is shown in Fig. 7. These boreholes were also used for seismic microzonation of Bangalore and other studies (Sitharam and Anbazhagan 2008a). For the purpose of general identification of soil layers in the study area, a generalized classification of soil has been attempted using borehole information and is given in Table 1. Table 1 shows that the western part of the city/study area has lesser overburden thickness when compared to other areas and consist of medium to dense soil. The eastern part of the city has a thicker overburden, consisting of loose to dense soil with fill material.

### N and $V_S$ Corrections

The $N$ values from field borelogs have been corrected for various corrections, such as (a) overburden pressure ($C_O$), (b) hammer energy ($C_E$), (c) borehole diameter ($C_D$), (d) presence or absence of liner ($C_L$), (e) rod length ($C_R$), and (f) fine content ($C_F$) (Seed et al. 1983; 1985; Skempton, 1986; Youd et al. 2001; Cetin et al. 2004; Pearce and Baldwin 2005). Corrected $N$ value, i.e., $(N_{10})$ is obtained using the following equation:

$$ (N_{10}) = N \times (C_N \times C_E \times C_D \times C_L \times C_F) $$

The SPT $N$ values recorded in the field increase with increasing effective overburden stress; hence overburden stress correction factor is applied (Seed and Idriss 1982). This factor is commonly calculated from the equation developed by Liao and Whitman (1986). However Kayen et al. (1992) has suggested the following equation, which limits the maximum $C_E$ value to 1.7 and provides a better fit to the original curve specified by Seed and Idriss (1982):

$$ C_E = 2.2/(1.2 + \alpha_v/P_a) $$

where:

- $\alpha_v$ = effective overburden pressure,
- $P_a = 100 \text{ kPa}$, and
- $C_E$ should not exceed a value of 1.7.

This empirical overburden correction factor is also recommended by Youd et al. (2001).

Another important factor that affects the SPT $N$ value is the energy transferred from the falling hammer to the SPT sampler. The energy ratio (ER) delivered to the sampler depends on the type of hammer, anvil, lifting mechanism, and the method of hammer release. Where energy measurements cannot be made, careful observation and notation of the equipment and procedures are necessary to estimate the $C_E$ value. The use of good-quality testing equipment and carefully controlled testing procedures will generally yield more consistent ERs. For liquefaction calculation, Yilmaz and Bagci (2006) took the $C_E$ value as 0.7 for the SPT hammer donut type, delivering 60% energy. In this study the delivered hammer energy is not measured, but the hammer used is similar to Yilmaz and Bagci (2006), so $C_E$ is taken as 0.7.

The other correction for borehole diameter, rod length, and sampling methods are modified from Skempton (1986). The correction
TABLE 1—Soil distribution in Bangalore.

<table>
<thead>
<tr>
<th>Layer</th>
<th>Soil Description</th>
<th>Depth (m)</th>
<th>Direction</th>
<th>Details of Sampling</th>
<th>SPT N Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>First layer</td>
<td>Silty sand with clay, 0–3 m</td>
<td>0.0</td>
<td>Northwest</td>
<td></td>
<td>1.5</td>
</tr>
<tr>
<td>Second layer</td>
<td>Medium to dense silty sand, 3–6 m</td>
<td>2.0</td>
<td>Southwest</td>
<td></td>
<td>1.5</td>
</tr>
<tr>
<td>Third layer</td>
<td>Weathered rock, 6–11 m</td>
<td>3.0</td>
<td>Northeast</td>
<td></td>
<td>1.5</td>
</tr>
<tr>
<td>Fourth layer</td>
<td>Hard rock, below the 11 m</td>
<td>4.5</td>
<td>Southeast</td>
<td></td>
<td>1.5</td>
</tr>
<tr>
<td>Fifth layer</td>
<td>Hard rock</td>
<td>6.0</td>
<td></td>
<td></td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>Weathered Rock</td>
<td>16.5</td>
<td></td>
<td></td>
<td>10.5</td>
</tr>
</tbody>
</table>

Note: * Sample not retrieved

FIG. 7—Typical borelog with the SPT N values.

TABLE 2—Typical N correction table for borelog.

<table>
<thead>
<tr>
<th>Borehole</th>
<th>Correction Factors for</th>
<th>Water Table=1.4 m/19-11-2005</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth (m)</td>
<td>Field N Value</td>
<td>Density (kN/m³)</td>
</tr>
<tr>
<td>1.50</td>
<td>19</td>
<td>20.00</td>
</tr>
<tr>
<td>3.50</td>
<td>28</td>
<td>20.00</td>
</tr>
<tr>
<td>4.50</td>
<td>26</td>
<td>20.00</td>
</tr>
<tr>
<td>6.00</td>
<td>41</td>
<td>20.00</td>
</tr>
<tr>
<td>7.50</td>
<td>55</td>
<td>20.00</td>
</tr>
<tr>
<td>9.00</td>
<td>55</td>
<td>20.00</td>
</tr>
<tr>
<td>10.50</td>
<td>100</td>
<td>20.00</td>
</tr>
<tr>
<td>12.50</td>
<td>100</td>
<td>20.00</td>
</tr>
</tbody>
</table>

Note: TS: Total stress; ES: Effective stress; C₅₀: Correction for overburden correction; (N₁₀₀): Corrected N value before correction for fine content; FC: Fine content; Δ(N₁₀₀): Correction for fine content; and (N₁₀₀)_{corr}: Corrected N value.
factors are listed by Robertson and Wride (1998). The borehole diameter correction factor of 1.05 for 150 mm borehole diameter is used. The rod length correction factor \(C_R\) is applied based on the length of the rod. Sampler correction factor \(C_S\) for the presence or absence of liner is taken as 1.0 for standard sampler. The corrected \(N\) value \((N_i)_{60c}\) is further corrected for fine content based on the revised boundary curves derived by Idriss and Boulanger (2004) for cohesionless soils as described below

\[
(N_i)_{60c} = (N_i)_{60} + \Delta(N_i)_{60}
\]

where:
\[
\Delta(N_i)_{60} = \exp\left[1.63 + \frac{9.7}{FC + 0.001} - \left(\frac{15.7}{FC + 0.001}\right)^2\right]
\]

Typical corrected \(N\) values for a borehole is shown in Table 2. Similarly SPT \(N\) corrected values with depth have been determined for all the boreholes.

Shear wave velocities are corrected for overburden stress using traditionally followed equation (Sykora 1987; Robertson et al. 1992; Andrus and Stokoe 2000; Youd et al. 2001; Juang et al. 2002; Andrus et al. 2004)

\[
V_s1 = V_s C_v
\]

\[
C_v = \left(\frac{P}{\sigma_{vo}'}\right)^{0.25}
\]

where:
\(V_s1\) = overburden stress corrected shear wave velocity and \(C_v\) = factor to correct measured shear wave velocity for overburden pressure.

A maximum \(C_v\) value of 1.4 is generally applied to \(V_s\) at shallow depth (Andrus and Stokoe 2000). The overburden stress corrected shear wave velocity \((V_s1)\) is evaluated for each layer, which is used to estimate the overburden stress corrected shear modulus \(G_{max}\).

\[
G_{max} = 29.12 N^{0.60}
\]

\[
R^2 = 0.88
\]

\[
G_{max} = 24.28 N^{0.55}
\]

\[
G_{max} = 19.43 N^{0.51}
\]

**Relation between \(G_{max}\) and Standard Penetration Test \(N\) Values**

The shear modulus at low strain level for soil layers has been determined using shear wave velocity from MASW and density from undistributed soil samples using Eq 8

\[
G = \rho V_s^2 = \frac{\gamma V_s^2}{g}
\]

where:
\(\rho\) = density measured from the undisturbed sample and \(V_s\) = shear wave velocity measured using the MASW testing. \(G_{max}\) has been evaluated for corresponding depth of \(N\) values in the respective locations. Typical \(G_{max}\) calculations for one location are given in Table 3.

The correlation between measured \(G_{max}\) (calculated from measured shear wave velocity and density of each layer) to the measured SPT \(N\) values is attempted. From the 38 sets of MASW and SPT testing points, about 215 pairs of \(N\) and \(G_{max}\) values have been used for the regression analysis.

To obtain the practical relationship between shear modulus and \(N\) values and to understand data matching, different combinations of corrected and uncorrected values were attempted, as discussed below.

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**TABLE 3—Typical \(G_{max}\) calculation table.**

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>(V_s) (m/s)</th>
<th>Density (g/cm(^3))</th>
<th>Shear Modulus (MN/m(^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–1.2</td>
<td>252</td>
<td>1.90</td>
<td>121</td>
</tr>
<tr>
<td>1.2–2.7</td>
<td>158</td>
<td>1.90</td>
<td>47</td>
</tr>
<tr>
<td>2.7–4.6</td>
<td>149</td>
<td>1.90</td>
<td>42</td>
</tr>
<tr>
<td>4.6–7.0</td>
<td>283</td>
<td>1.90</td>
<td>152</td>
</tr>
<tr>
<td>7.0–10</td>
<td>343</td>
<td>1.90</td>
<td>224</td>
</tr>
<tr>
<td>10.0–13.7</td>
<td>328</td>
<td>1.90</td>
<td>204</td>
</tr>
<tr>
<td>13.7–18.4</td>
<td>386</td>
<td>2.00</td>
<td>298</td>
</tr>
<tr>
<td>18.4–24.2</td>
<td>508</td>
<td>2.00</td>
<td>516</td>
</tr>
<tr>
<td>24.2–31.4</td>
<td>582</td>
<td>2.20</td>
<td>745</td>
</tr>
<tr>
<td>31.4–39.3</td>
<td>804</td>
<td>2.20</td>
<td>1422</td>
</tr>
</tbody>
</table>

---

**FIG. 8—Shear modulus versus measured/uncorrected SPT \(N\) values.**
Relation between Uncorrected Values

Correlation between measured values of SPT $N$ and shear modulus $(G_{\text{max}})$ is presented in Fig. 8. The regression equation between $G_{\text{max}}$ and $N$ is given below

$$G_{\text{max}} = 24.28N^{0.55}$$  \hspace{1cm} (9)

where:

- $G_{\text{max}}$ = low strain measured shear modulus in MN/m$^2$
- $N$ = measured SPT $N$ value.

Figure 8 also shows the actual data and fitted equation with ±20% error bars. The best fit equation has the regression coefficient $R^2$ squared value of 0.88. In addition regression equations with 95% confidence interval are shown in Fig. 8. The 95% confidence bands enclose the area that one can be 95% sure of the true curve. It gives a visual sense of how well the data defines the best fit curve (Motulsky 2008). Regression equations corresponding to 95% confidence intervals are given in Eqs 10 and 11, respectively

$$G_{\text{max}} = 29.17[N_{\text{60}}]^{0.57} \quad \text{R}^2 = 0.861$$  \hspace{1cm} (9)

$$G_{\text{max}} = 17.12[N_{\text{60cs}}]^{0.69} \quad \text{R}^2 = 0.858$$  \hspace{1cm} (10)

Relation between Corrected $N$ and Uncorrected $G_{\text{max}}$ Values

To study the difference between corrected and uncorrected SPT $N$ values in the regression equation, many combinations of plots were generated. Figure 9 shows the correlation between (a) corrected $N$ values and measured modulus and (b) corrected $N$ values with fine content correction and measured shear modulus. In the first case corrected $N$ values are estimated excluding fine content correction factor according to Eq 2, and in the second case corrected $N$ values are estimated including fine content correction factor according to Eq 4. The first one gives slightly higher $R^2$ value (0.86) when compared to the second one ($R^2 = 0.858$). The data range in the first case $[N_{\text{60}}]$ is distributed from 2 to about 90, but in the second case, it $[N_{\text{60cs}}]$ is distributed from 7 to about 90. For the purpose of developing the regression equation, any one of the corrected $N$ value without or with fine content correction ($[N_{\text{60}}]$ or $[N_{\text{60cs}}]$) may be used. This is so because the corrected $N$ values without or with

$$G_{\text{max}} = 19.43N^{0.51} \quad \text{lower side of 95% confidence interval}$$  \hspace{1cm} (11)
The developed regression equations for the corrected \( N \) values without or with considering fine content correction are given below: Without fine content correction

\[
G_{\text{max}} = 29.17 [(N_1)_{60}]^{0.57} \quad (12)
\]

with fine content correction

\[
G_{\text{max}} = 17.12 [(N_1)_{60cs}]^{0.69} \quad (13)
\]

**Relation between Corrected \( N \) and Corrected \( G_{\text{max}} \) Values**

The overburden stress corrected shear modulus has been evaluated using traditional applied \( V_s \) correction factor given in Eq 7. Figure 10(a) and 10(b) shows the corrected shear modulus (\( G_{\text{max} 1} \)) versus corrected \( N \) values of \( (N_1)_{60} \) and \( (N_1)_{60cs} \). It was found that the regression fit is poor and gives lower \( R^2 \) values when compared to corrected \( N \) and uncorrected \( G_{\text{max}} \) values. Also similar to the above results, the corrected \( N \) values without or with fine content correction gives similar fitted equation and \( R^2 \) values. The regression equation for the corrected shear modulus (\( G_{\text{max} 1} \)) and corrected \( N \) values without or with considering fine content correction is given below: Without fine content correction

\[
G_{\text{max} 1} = 46.04 [(N_1)_{60}]^{0.46} \quad R^2 = 0.68 \quad (14)
\]

with fine content correction

\[
G_{\text{max} 1} = 28.62 [(N_1)_{60cs}]^{0.56} \quad R^2 = 0.68 \quad (15)
\]

To trace out the problems, an attempt is made to use similar overburden stress correction factor applied to \( N \) values. The overburden stress corrected shear modulus of each layer has been evaluated using the correction factor given in Eq 3. Figure 11(a) and 11(b) shows the corrected shear modulus (\( G_{\text{max} 2} \)) versus corrected \( N \) values of \( (N_1)_{60} \) and \( (N_1)_{60cs} \). The regression fit is very poor and gives lower \( R^2 \) values when compared to corrected \( N \) and corrected \( G_{\text{max} 1} \) values. The regression equation for the corrected shear modulus (\( G_{\text{max} 2} \)) and corrected \( N \) values without or with fine content correction is given below: Without fine content correction

\[
G_{\text{max} 2} = 61.07 [(N_1)_{60}]^{0.37} \quad (16)
\]

With fine content correction
Corrected shear modulus $G_{max2}$

$G_{max2} = 61.07[(N_1)_{60}]^{0.37}$

$R^2 = 0.47$

Corrected shear modulus $G_{max2}$

$G_{max2} = 39.83[(N_1)_{60_c}]^{0.47}$

$R^2 = 0.48$

$\pm 20\%$ Error bar

FIG. 11—Corrected shear modulus ($G_{max2}$) versus corrected SPT $N$ values.

Kramer (1996) referring
Imai and Tonouchi, (1982)

$G_{max} = 325(N_{60})^{0.68}$

Imai and Tonouchi, (1982)

$G_{max} = 144N^{0.68}$

FIG. 12—Comparison of shear modulus equations.
Corrected N without and with fine content correction versus observed N values.

\[ (N_{160}) = 1.02(N)^{0.88} \]
\[ R^2 = 0.95 \]

\[ (N_{160cs}) = 2.17(N)^{0.74} \]
\[ R^2 = 0.95 \]

Comparison of shear modulus versus N values developed in this study.

\[ G_{max} = 29.17((N_{160})^{0.57} \]
\[ G_{max} = 24.28N^{0.55} \]
\[ G_{max} = 17.12((N_{160cs})^{0.69} \]
From this study, it is clear that the correlation between the normalized shear modulus and \( N \) does not give a better regression equation even though the data are same. It is necessary to review the traditionally used overburden stress correction factor for \( V_s \). The traditional shear wave velocity correction factors may be revised in future based on these data or by including more data if available.

**Results and Discussions**

To validate \( N \) versus \( G_{\text{max}} \) relation presented in this study, it has been compared with existing relations available in the literature. Many regression equations between \( N \) \((N_1)_{60}\) and \((N_1)_{60cs}\) with \( V_s \) and \( X \) are available in the literature for different soils by different researchers, but limited regression equations are available for \( N \) versus \( G_{\text{max}} \). Popularly used correlations are given by Ohta and Goto (1978) [subsequent revision of data are presented by Seed et al. (1986)] and Imai and Tonouchi (1982). Seed et al. (1986) developed regression \( V_s \) relation by considering \( N_{60} \) (\( N \) values measured in SPT test delivering 60% of theoretical free fall energy to the drill rods) and depth of soil in feet (\( D \)). Then by assuming unit weight (\( \gamma \)) of 1.92 \( \text{g/cm}^3 \) (120 pound per cubic foot), the \( G_{\text{max}} \) was calculated according to Eq 8, and \( G_{\text{max}} \) relation was developed. So comparing present equation with the Seed et al. (1986) relation may not be appropriate. Hence, a comparison has been made with the study similar to the present study by Imai and Tonouchi (1982). Imai and Tonouchi (1982) developed \( N \) versus \( G_{\text{max}} \) relation using the average \( N \) values from single velocity layers, and also, \( N \) values of above 50 and below 1 are substituted for the number of blows required to achieve a penetration depth of 30 cm from actual amount of penetration achieved at 50 blows. Imai and Tonouchi (1982) used a large number of data from different age and soil types and presented the \( N \) values and shear modulus relation separately according to geology and soil type and based on the overall data. The original equation proposed by Imai and Tonouchi (1982) is

\[
G_{\text{max}} = 29.17[N_1]_{60}^{0.57}
\]

\[
G_{\text{max}} = 24.28N^{0.55}
\]

\[
G_{\text{max}} = 17.12[N_1]_{60cs}^{0.69}
\]

\[
+G_{\text{max}} = 144(N)^{0.68}
\]

The same equation was reproduced in Kramer (1996) for the corrected \( N \) values for sand, which is as follows (Kramer (1996) referring Imai and Tonouchi (1982)):

\[
G_{\text{max}}(\text{kips/ft}^2) = 325[N_{60}]^{0.68}
\]

Figure 12 shows both the Eqs 18 and 19 using uncorrected values of \( N \). In order to compare equations developed in our study, it is assumed that Eq 19 based corrected \( N \) values (\( N_{60} \)) as highlighted in the work of Kramer (1996). To compare the regression equations in a single plot, a relation between corrected \( N \) values \( [(N_1)_{60}] \) or \( (N_1)_{60cs} \) to measured \( N \) values was developed, which is shown in Fig. 13. The best fit regression equation for the corrected \( N \) values without and with considering fine content correction \( [(N_1)_{60}] \) or \( (N_1)_{60cs} \) and observed \( N \) values are given below

\[
(N_1)_{60} = 1.02(N)^{0.88}
\]

\[
(N_1)_{60cs} = 2.17(N)^{0.74}
\]

Figure 14 shows a comparison of the Eqs 9, 12, and 13 using the above Eqs 20 and 21. In Fig. 14, horizontal line gives uncorrected or corrected SPT \( N \) values based on the equation. If SPT \( N \) value of \( X \) is uncorrected for Eq 9, the same \( X \) is corrected \( N \) value without fine content correction for Eq 12 and corrected \( N \) value with fine content correction for Eq 13. This similar explanation also applies for Figs. 15 and 16. From Figs. 12 and 14, it is clear that corrected \( N \) values without fine content corrections yield a higher shear modulus (\( G_{\text{max}} \)) against measured \( N \) values. The trends of the results for \( N \) values without considering fine content correction is similar to the one presented in Fig. 12. From Fig. 14, \( G_{\text{max}} \) equations developed using corrected \( N \) considering fine content correction (Eq 13) matches with \( G_{\text{max}} \) equation using measured \( N \) values (Eq 9) up to the \( N \) values of 20 and \( G_{\text{max}} \) equation using corrected \( N \) without considering fine content correction (Eq 12) for \( N \) values above 30. For the \( N \) values between 20 and 30, Eq 13 lies in between Eqs 9 and 12.

Figure 15 shows the comparison of equations developed in this study with Eq 18 presented by Imai and Tonouchi (1982). From Fig. 15, the developed relation between measured \( N \) values and
measured shear modulus matches well with the Eq 18 beyond the $N$ value of 10; this may be attributed to soil type and number of data. In Imai and Tonouchi (1982) most of the data are below the $N$ value of 10 (for clay and sand). However, in this study, soil is a mixture of sandy silt with clay and does not have many data for $N<10$. Equation 13 shows a trend similar to that of Imai and Tonouchi (1982) equations. Figure 16 shows a comparison of equations developed in this study for corrected values, with Eq 19 presented by Kramer (1996). Shear modulus values have been evaluated considering $(N_1)_{60}$ and $(N_1)_{60cs}$ using Eq 19. From Fig. 15, regression relations developed in this study between corrected $N$ values $[(N_1)_{60}$ and $(N_1)_{60cs}]$ and measured shear modulus (Eqs 11 and 12) have similar trends. So in the equation given in Kramer (1996), the corrected $N$ values might have been used. Developed equation based on the corrected $N$ values match well with those of Kramer (1996). The correlations developed in this study used the measured $SPT\ N$ values of up to 100 (refusal), whereas Imai and Tonouchi (1982) had used a measured $N$ value of up to 50. Also, the proposed correlation is unique, independent of the lithology, soil grading, age, cementation, etc. The developed equations in this study can be directly used for soil type “silty sand or sandy silt with less clay content;” however for the important structures, the best way is to measure $G_{\text{max}}$ directly using in situ seismic tests.

### Conclusions

Regression relation between SPT $N$ and $G_{\text{max}}$ values have been developed using 215 pairs of SPT $N$ and $G_{\text{max}}$ from geotechnical borelogs and geophysical MASW data. The regression equation using measured values gives best fit and $R^2$ values when compared to the corrected $N$ and corrected $G_{\text{max}}$. The regression relation between corrected $N$ values without considering fine content correction $[(N_1)_{60}]$ or with considering fine content correction $[(N_1)_{60cs}]$ and $G_{\text{max}}$ are also giving similar $R^2$ values. In the relation between corrected $N$ values and measured shear modulus, any one of the corrected $N$ values (without or with considering fine content correction) can be used for regression analysis. The relation between corrected $N$ and $G_{\text{max}}$ shows poor regression relation for the same data. The traditional overburden stress correction factors applied for shear wave velocity need a relook. The traditional shear wave velocity correction factors may be revised in the future based on these data or including more data if available. The proposed relations are comparable with the existing relations similar to this study. Existing relations were developed with many assumptions during the developing stage of geotechnical earthquake engineering (GEE). These relations have to be reviewed with present knowledge of GEE and may be updated and reproduced in the future. The developed equation between $N$ and $G_{\text{max}}$ is more suitable for residual soils (i.e., silty sand or sandy silt) with less percentage of clay content.

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