Time Domain Seismic Reliability Evaluation of Dynamic Systems Using a Hybrid Approach

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Collaborators

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- **Prof. A. Reyes-Salazar** (nonlinear, 3-D stress-based FEM, Seismic)
- **Prof. J. Huh** (nonlinear, 3-D stress-based FEM, dynamic loading in time domain, steel frames)
- **Dr. S.Y. Lee** (nonlinear, 3-D stress-based FEM, dynamic loading in time domain, steel frames with concrete shear walls)
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Motivation

- Safety evaluation using realistic behavior of structures and loadings.
- Need for efficient & accurate reliability analysis method of nonlinear structures, similar to deterministic community.
- Finite element formulation is very desirable; deterministic community is very familiar with it and uses it routinely.

Motivation

- Consideration of uncertainties in dynamic loading along with gravity loadings

- Random vibration approach – we need to find an alternative to it since it will not satisfy the deterministic community

- Time history analysis

Realistic Performance Evaluation

- Three Major Sources of Uncertainty
  - Environmental
    - Load-related variables
  - Structural
    - Resistance-related variables
      - Structural geometry
      - Material properties
      - Boundary conditions

Realistic Performance Evaluation (cont.)

- **Modeling**
  - **Support conditions**
    - **Fixed**
    - **Pinned**
    - **Roller**

Supports are partially fixed with different rigidities

- **Connection conditions**
  - **Fully restrained (FR)**
  - **Partially restrained (PR)**

Different assumptions have significant design implications.

Structural Design

- the allowable stress design (ASD) concept
- the ultimate strength design method
- the load and resistance factor design (LRFD) concept
- the performance based design (PBD) guidelines

Performance-Based Design Guidelines

- PBD guidelines are expected to be in the next generation code.

- Engineers will be asked to design structures satisfying performance requirements

- In this approach, instead of return period or partial safety factors concept, the probability requirements will be set to satisfy the performance requirements.

- At present, we do not have necessary reliability evaluation technique that will satisfy the professional requirements or the deterministic community.

Performance-Based Design Guidelines

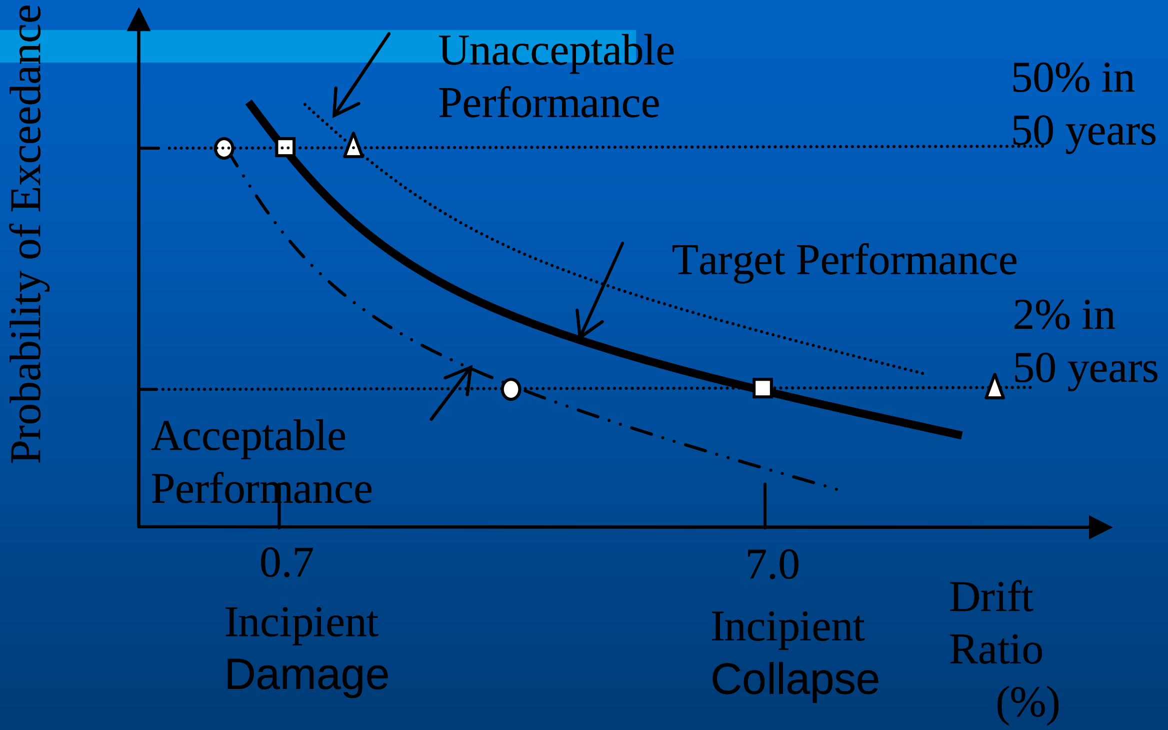


Figure 1. Bi-level acceptance criteria in terms of 50-year limit state probability

Reliability Analysis Method

Explicit Limit State

- Based on Safety Index concept
  1. Mean value first-order second-moment (MVFOSM) method.
  2. Hasofer-Lind "Generalized safety index" method.
  3. First-order reliability method (FORM).
  4. Second-order reliability method (SORM).
- Based on probability of failure concept
  1. Distribution-fitting method.
  2. Monte Carlo simulation (MCS).

Reliability Analysis Method (cont.)

- **Implicit Limit State**

1. Monte Carlo simulation (MCS).
2. Stochastic Finite Element Method (SFEM).

Justification for SFEM Concept

1. **Approximation of the limit state function Method**

- **Second order polynomial approximation in the neighborhood of the design point (Wu, Wu and Wirsching; they did not demonstrate its use for the FEM formulation)**

- **Response surface method**

- **Polynomial approximation to the limit state (Wu, Burnside, Dominquez)**

Justification for SFEM Concept

2. **Perturbation methods**

- First or Second Order Taylor Series
- Neumann Expansion
- Karhunen-Loeve Orthogonal Expansion
- Polynomial Chaos

➢ These methods generally do not need the information on distribution

➢ Acceptable results are generally obtained for small random fluctuations

3. **Reliability Approach**

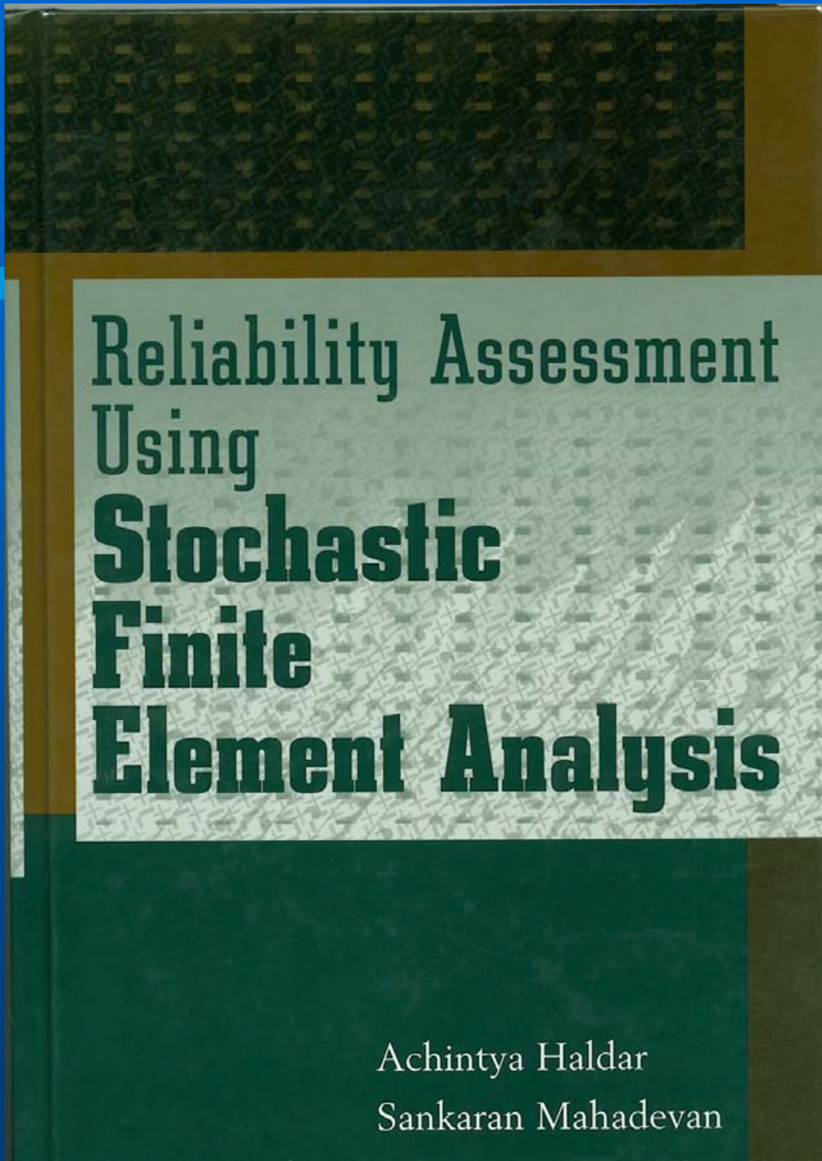
- Limit state concept– FORM/SORM approach

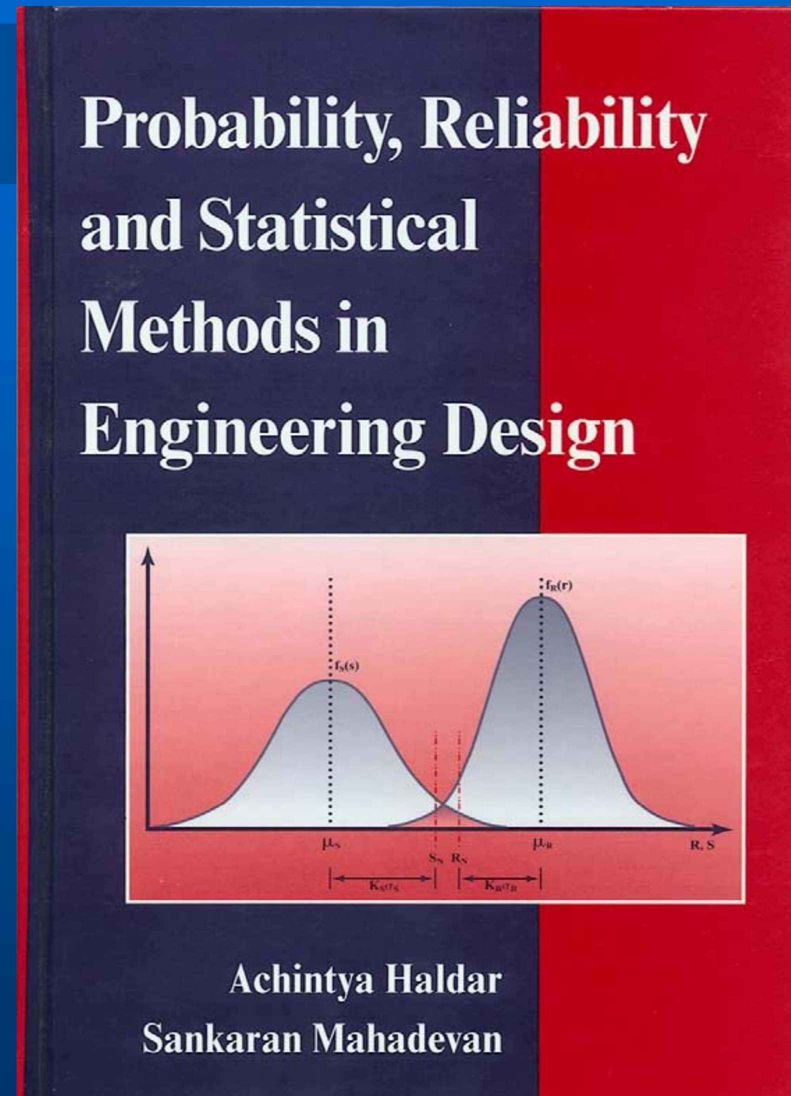
SFEM Concept

- **Efficient deterministic FEM is necessary for efficient algorithm**

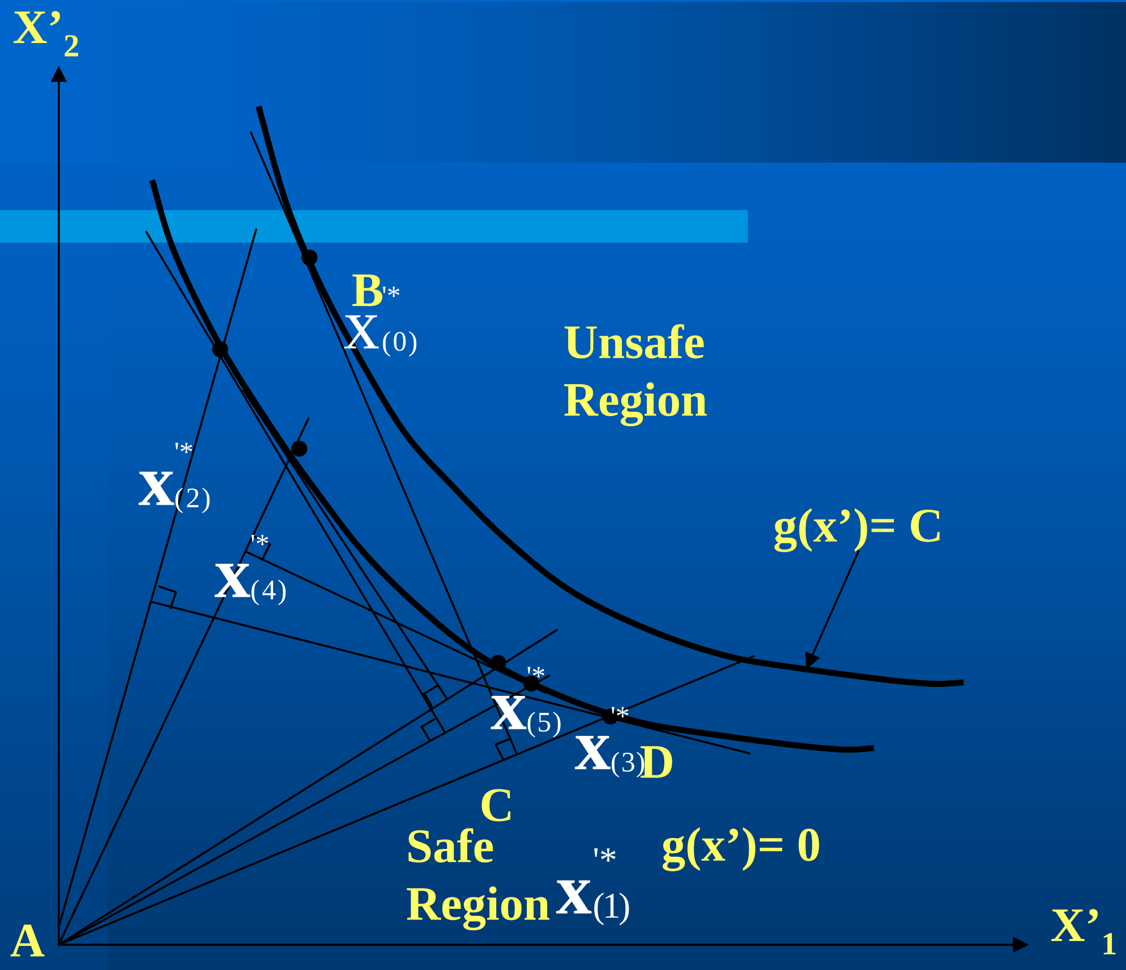
- **Displacement-based FEM (commonly used in most commercially available computer program)**

- **Stress-Based FEM (used in our study)**


Reliability Assessment
Using
Stochastic
Finite
Element Analysis
Achintya Haldar
Sankaran Mahadevan


Probability, Reliability
and Statistical
Methods in
Engineering Design
R, S
Achintya Haldar
Sankaran Mahadevan

Limit State Concept - FORM



Note: A number in parenthesis indicates iteration number

Basic Steps in FORM

Limit state: *G(X1, X2, …. Xn) = 0*

1. Transform variables X into the standard normal space Y so that the elements of Y are statistically independent, standard normal variables.

2. Choose a checking point yi. Use FEM to obtain *G*(yi) and $\nabla G(\mathbf{y}_i)$

3. Calculate the unit vector $\alpha_i = -\dfrac{\nabla G(\mathbf{y}_i)}{|\nabla G(\mathbf{y}_i)|}$

4. Find the next checking point yi+1 = $\left[\mathbf{y}_i^t \alpha_i + \dfrac{G(\mathbf{y}_i)}{|\nabla G(\mathbf{y}_i)|}\right]\alpha_i$

5. Repeat Steps 2, 3, and 4 until the point y* is obtained, at which the probability density in the standard normal space is the largest (design point).

6. Obtain the reliability index $\beta = \sqrt{\mathbf{y}^t\mathbf{y}}$ and estimate the failure probability using the approximate linear failure surface $P_f = \phi(-\beta)$

FORM in the context of SFEM

1. Define limit state as $G(\mathbf{x}, \mathbf{u}, \mathbf{s}) = 0$

X– a set of **basic random** variables pertaining to a structure (e.g., loads, material properties and structural geometry)

u – a set of **displacements** in the limit state function

s – a set of **load effects** (except the displacements, such as internal forces)

➤ The displacement u = QD, where D is the **global displacement** vector and Q is a transformation matrix

To implement the algorithm and assuming the limit state equation has a general form of G(X,U,S) = 0, the gradient of the limit state function in the standard normal space can be derived as

$$\nabla G(y) = \left[\frac{\partial G}{\partial s} J_{s,x} + (Q\frac{\partial G}{\partial u} + \frac{\partial G}{\partial s} J_{s,D}) J_{D,x} + \frac{\partial G}{\partial x} \right] J^{-1}_{y,x}$$

where $J_{i,j}$'s are the Jacobians of transformation (e.g., $J_{s,x} = \partial s / \partial x$)

FORM in the context of SFEM

- The essential numerical aspects of SFEM were just discussed in the evaluation of the three partial derivatives and four Jacobians in the above equation. The evaluation of these quantities will depend on the problem under consideration (linear or nonlinear, 2D or 3D, etc.) and the performance functions used.
- This is not simple.

Performance Functions

1. Serviceability Limit State

- Overall lateral displacement of the structure
- Code-specified allowable value

➢ For the serviceability criterion, the limit state function is represented as

$$g(\mathbf{x}, \mathbf{u}, \mathbf{s}) = 1.0 - \frac{\delta}{\delta_{limit}}$$

Where δ is the calculated displacement component and δ_{limit} is the prescribed maximum value of the displacement component.

Performance Functions (Cont.)

2. Strength Limit State

- Behavior of local structural elements
- Combined effect of axial load and bending moment
- Dynamic effect in interaction equation

According to the American Institute of Steel Construction's (AISC's) Load and Resistance Factor Design (LRFD) design guidelines, the strength performance criteria for 2-D steel frame members can be defined as

Performance Functions

$$g(x,u,s) = 1.0 - \frac{P_u}{P_n} - \frac{8}{9}\left(\frac{M_{ux}}{M_{nx}}\right); \qquad \text{if } \frac{P_u}{\phi P_n} \geq 0.2$$

$$g(x,\ u,\ s) = 1.0 - \left(\frac{P_u}{2P_n} + \frac{M_{ux}}{M_{nx}}\right); \qquad \text{if } \frac{P_u}{\phi P_n} < 0.2$$

P_u = **required tensile and compresssive strength**

P_n = **nominal tensile and compressive strength**

M_{ux} = **required flexural strength**

M_{nx} = **nonminal flexural strength**

P_u and M_{ux} **are unfactored load effects**

Nonlinear Time Domain Dynamic Problems – Implicit Limit States

- Are different for each time increment
- Are functions of time

Direct Monte Carlo Simulation (MCS)

- 100,000 cycles of simulation for a simple **one-bay two-story structure** ⇒ approximately 22 hrs 45 mins using super computer, ORIGIN 2000
- For a real complex 3-dimensional structure, several weeks or months even using super computer
- Astronomical amount of time if PC is used

➢ Response Surface Method (RSM)

- Raw RSM cannot be used for the dynamic reliability analysis
  - Needs to be modified
  - Needs to be incorporated with other schemes

- **A hybrid type of method is necessary for the dynamic reliability analysis in time domain**

Classical RSM

- For its efficient use, needs to be generated in the failure region
- Also, it fails to incorporate information on distribution even when it is available
- Integration with FORM/SORM can incorporate the desirable features

A Unified Dynamic Reliability Analysis Algorithm

1. Assumed stress-based FEM

   To evaluate dynamic response in time domain

2. First-order reliability method (FORM)

3. Response surface method (RSM)

4. Iterative linear interpolation scheme

Response Surface Method (RSM)

To obtain explicit limit state function

- by approximating the original implicit limit state By a simple polynomial
- in terms of basic random variables

- **Important elements of RSM**

1. Degree of polynomials
2. Experimental design
3. Experimental region

Degree of Polynomial

Nonlinear time domain seismic response

⇒ Second order polynomial without cross terms

$$\hat{g}(\mathbf{X}) = b_0 + \sum_{i=1}^{k} b_i X_i + \sum_{i=1}^{k} b_{ii} X_i^2$$

Number of coefficients to be determined; *p = 2k + 1*

⇒ Second order polynomial with cross terms

$$\hat{g}(\mathbf{X}) = b_0 + \sum_{i=1}^{k} b_i X_i + \sum_{i=1}^{k} b_{ii} X_i^2 + \sum_{i=1}^{k-1} \sum_{j>i}^{k} b_{ij} X_i X_j$$

Number of coefficients to be determined; *p = (k+1)(k+2)/2*

Experimental Design

- **Classical Design**
  - **Factorial Design**
    - **Central Composite Design (CCD)**
      - **Applicable only for a full second order**
      - **polynomial with cross terms**
      - **Number of sampling points = 2k + 2k + 1**
      - **Regression analysis**
      - **Orthogonality, rotatability, ANOVA**

      - **Accurate but inefficient when *k* is large**

Experimental Design

- **Saturated Design (SD)**

  - **Applicable for both types of polynomial**

  - **Number of sampling point = number of coefficients in the polynomial**

  - **Efficient for large *k***

  - **May not cover sample space between axes (relatively inaccurate)**

  - **Lacks statistical properties**

Response Surface Models

Model 1 - SD using second order polynomial without cross terms

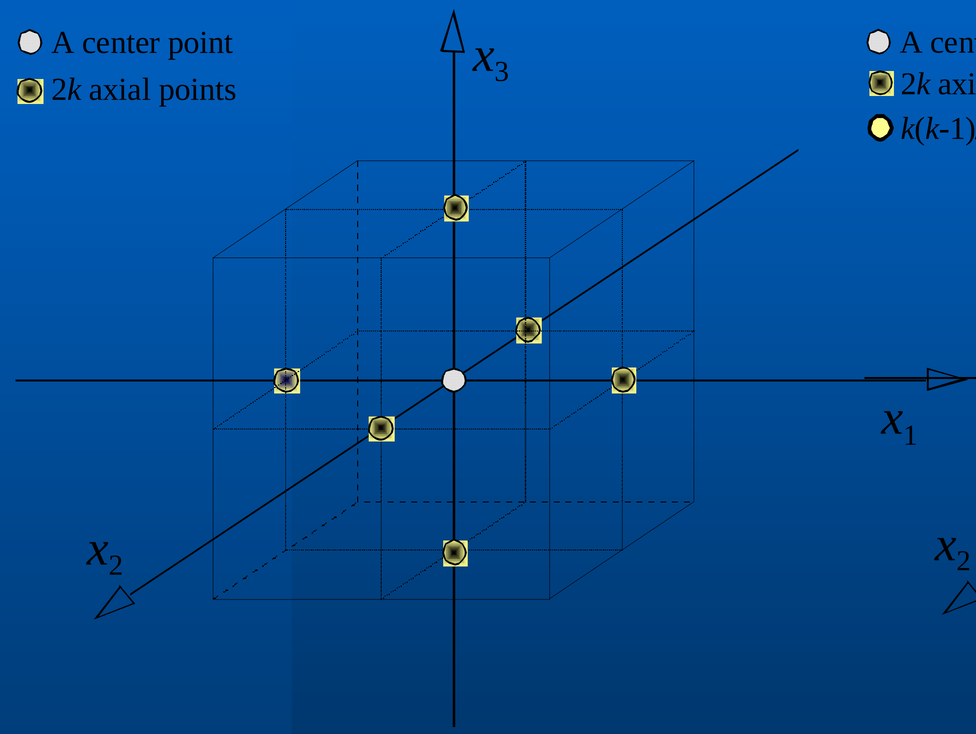
Model 2 - SD using a full second order polynomial

Model 3 - CCD using a full second order

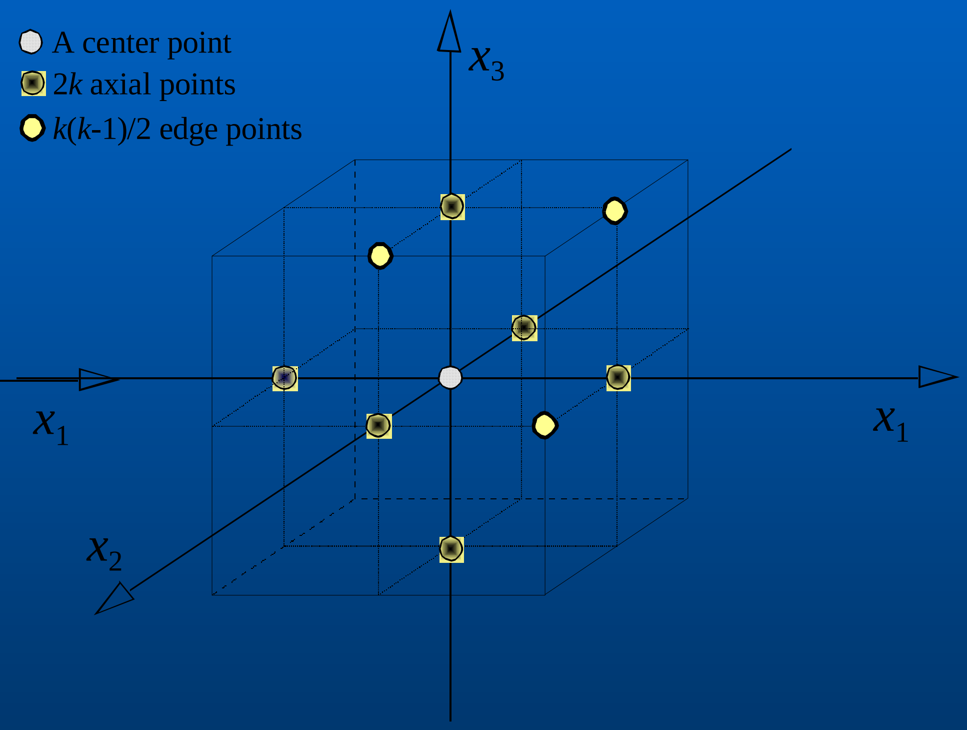
Four Schemes for the Proposed Algorithm

Schemes (1)	Intermediate Iteration (2)	Final Iteration (3)
Scheme ①	Model (1)	
Scheme ②	Model (3)	
Scheme ③	Model (1)	Model (2)
Scheme ④	Model (1)	Model (3)

Response Surface Models

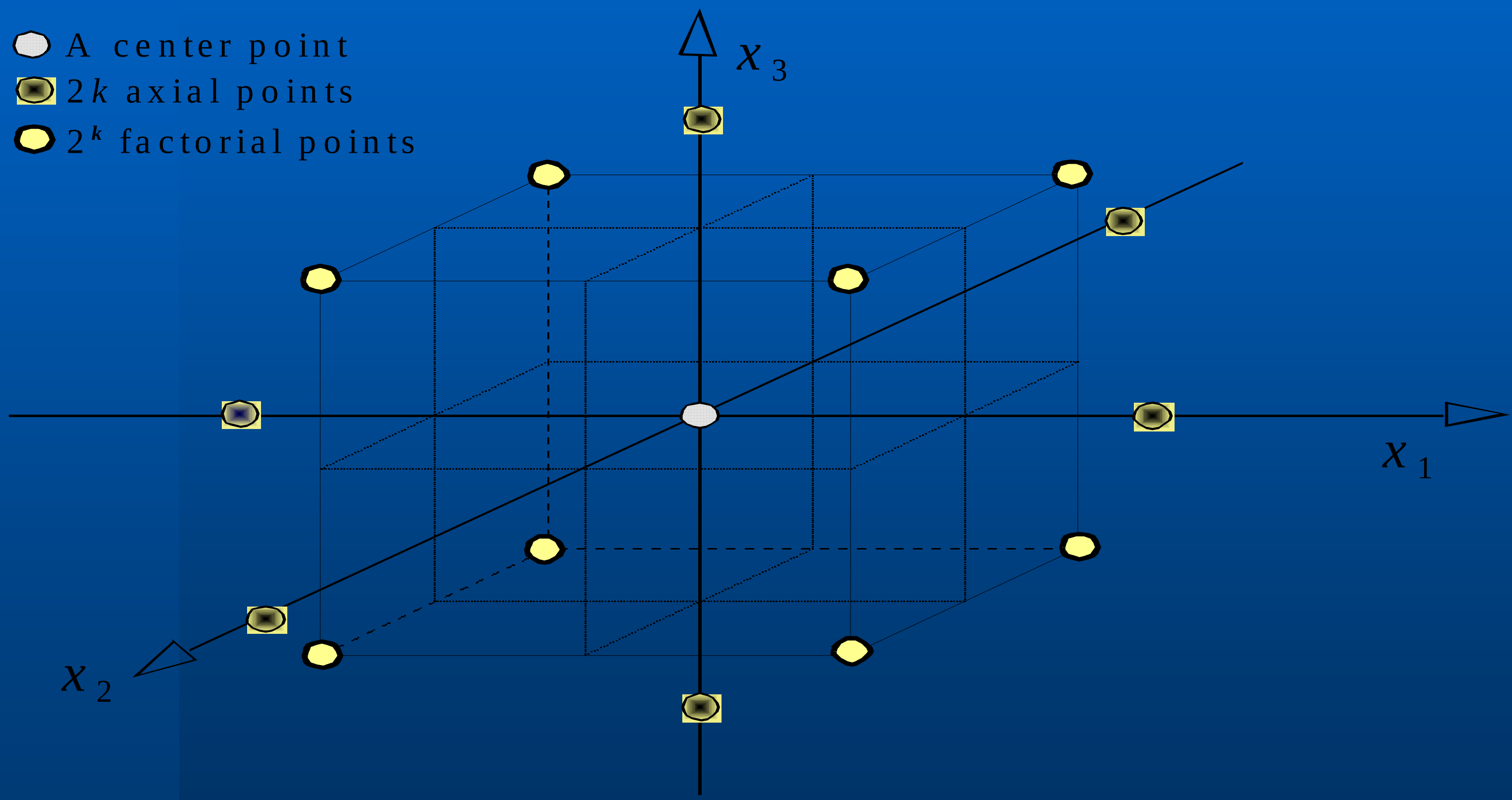


Model (1) in the coded variable space for *k* = 3



Model (2) in the coded variable space for *k* = 3

Response Surface Models



Model (3) in the coded variable space for k = 3

Comparison of the Three Models

Model	Number of Coefficients		*k*	Number of Sample Points	
Model (1)	$p = 2k + 1$	7	3	7	$N = 2k + 1$
		17	8	17	
		25	12	25	
Model (2)	$p = \frac{(k+1)(k+2)}{2}$	10	3	10	$N = \frac{(k+1)(k+2)}{2}$
		45	8	45	
		91	12	91	
Model (3)		10	3	15	$N = 2^k + 2k + 1$
		45	8	273	
		91	12	4142	

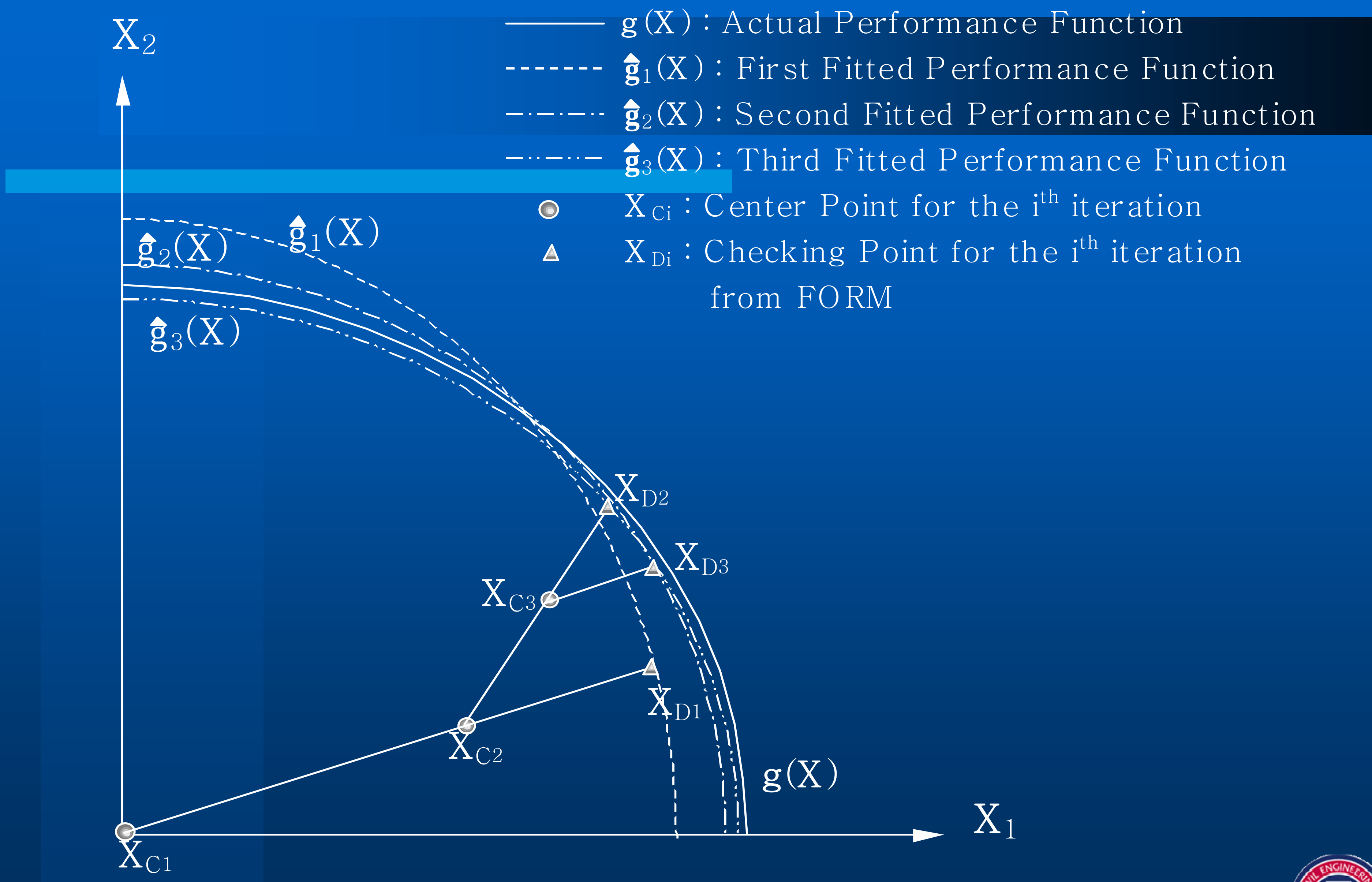
Experimental region

Determination of Center Point

Iterative Linear Interpolation Scheme

$$\mathbf{x}_{C_{i+1}} = \mathbf{x}_{C_i} + \left(\mathbf{x}_{D_i} - \mathbf{x}_{C_i}\right)\frac{G\left(\mathbf{x}_{C_i}\right)}{G\left(\mathbf{x}_{C_i}\right) - G\left(\mathbf{x}_{D_i}\right)} \quad if\ G\left(\mathbf{x}_{D_i}\right) \geq G\left(\mathbf{x}_{C_i}\right)$$

$$\mathbf{x}_{\mathrm{C}_{\mathrm{i+1}}} = \mathbf{x}_{\mathrm{D}_{\mathrm{i}}} + \left(\mathbf{x}_{\mathrm{C}_{\mathrm{i}}} - \mathbf{x}_{\mathrm{D}_{\mathrm{i}}}\right)\frac{\mathrm{g}\left(\mathbf{x}_{\mathrm{D}_{\mathrm{i}}}\right)}{\mathrm{g}\left(\mathbf{x}_{\mathrm{D}_{\mathrm{i}}}\right) - \mathrm{g}\left(\mathbf{x}_{\mathrm{C}_{\mathrm{i}}}\right)} \quad \text{if } \mathrm{g}\left(\mathbf{x}_{\mathrm{D}_{\mathrm{i}}}\right) < \mathrm{g}\left(\mathbf{x}_{\mathrm{C}_{\mathrm{i}}}\right)$$


g(X) : Actual Performance Function
ĝ1(X) : First Fitted Performance Function
ĝ2(X) : Second Fitted Performance Function
ĝ3(X) : Third Fitted Performance Function
XCi : Center Point for the ith iteration
XDi : Checking Point for the ith iteration from FORM
X2
X1
ĝ1(X)
ĝ2(X)
ĝ3(X)
g(X)
XD1
XD2
XD3
XC1
XC2
XC3

Iterative Steps

- **the initial center point is assumed to be the mean values of the random variables for the first iteration.**

- **Using nonlinear FEM, the responses are calculated at the experimental sampling points – without or with cross terms**

- **A limit state function (LSF) is thus generated in terms of *k* basic random variables.**

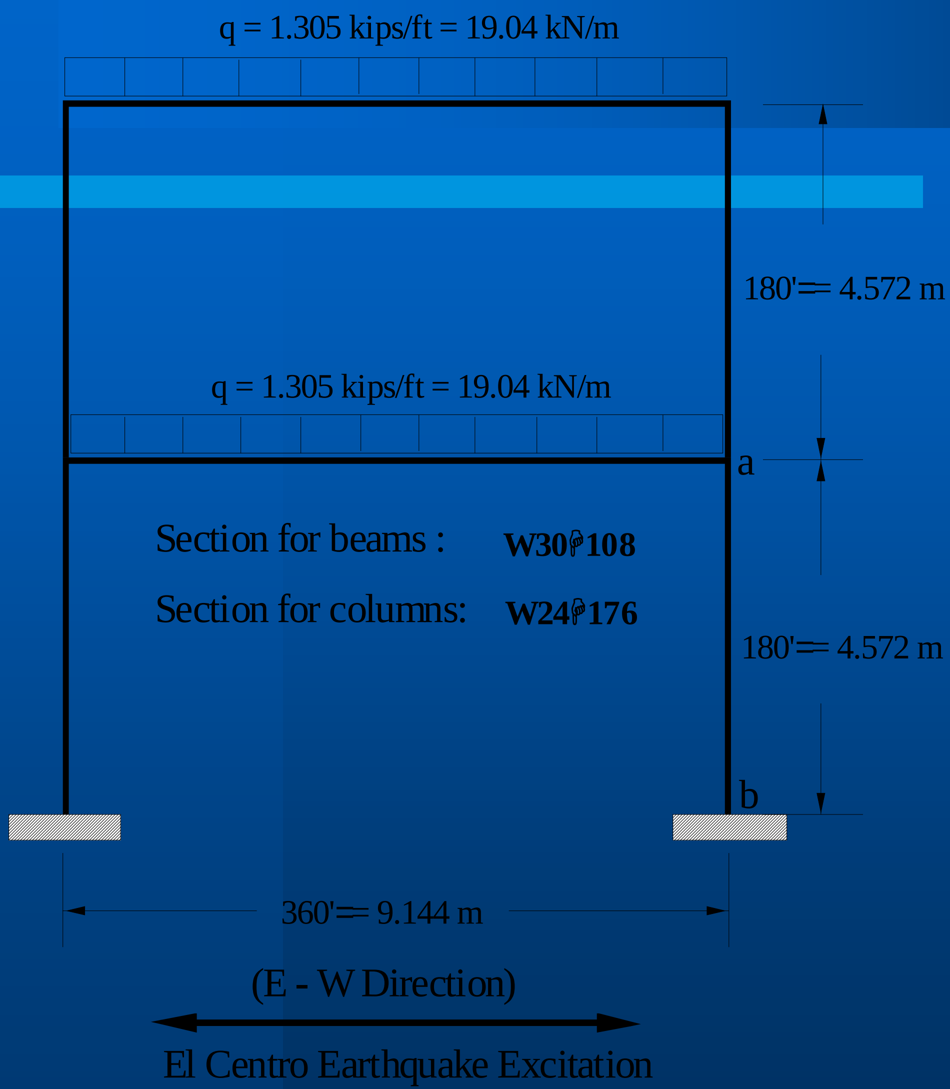
- **Using the explicit expression for the (LSF) and FORM, β, the corresponding coordinates of the checking point, and direction cosines are obtained for each random variable.**

Iterative Steps (cont.)

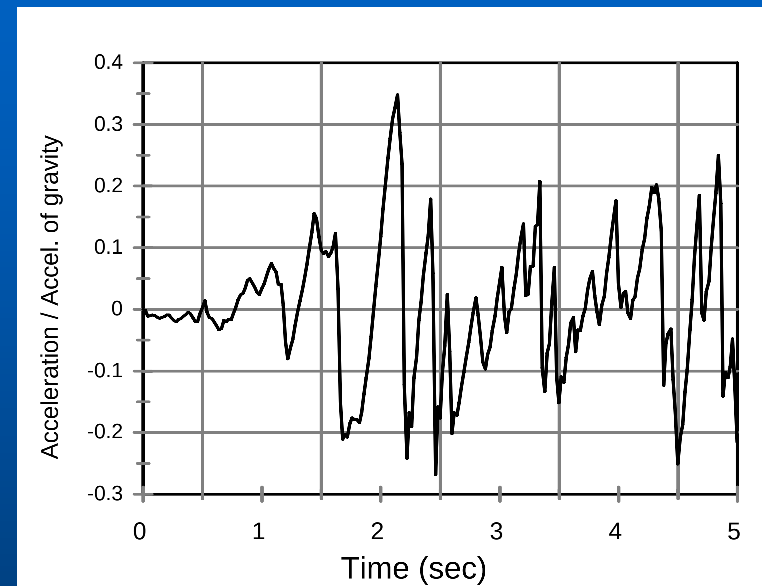
- The coordinate of the new center point is obtained by applying the linear interpolation scheme.
- The updating of the center point continues until it converges to a predetermined tolerance level.
- In the final iteration, the information on the most recent center point is used to formulate the final response surface using either saturated design with a full second order polynomial or CCD with a full second order polynomial.

Examples to show the elegance of the method

- Two story steel frame excited by El Centro earthquake time history
- Thirteen story steel frame excited by Northridge earthquake for verification
- Steel frame without and with shear walls
- Steel frames with pre- and post-Northridge connections
- Three-story three-bay frame with different levels of shear walls



Two-Story Steel Frame Structure



El Centro Earthquake Excitation Time History for 5.12 seconds (E-W)

Statistical Description of the Random Variables for Example 1

Limit State		Serviceability Limit State				Strength Limit State	
Random Variable (1)	Mean Value (2)	Case 1: 9 R.V.		Case 2: 7 R.V.			
		COV (3)	Dist. (4)	COV (5)	Dist. (6)	COV (7)	Dist. (8)
E (kN/m^2)	1.9994×10^8	0.06	LN	0.06	LN	0.06	LN
A^b (m^2)	2.045×10^{-2}	0.05	LN	0.05	LN	-	-
I^b_x (m^4)	1.861×10^{-3}	0.05	LN	0.05	LN	0.05	LN
Z^b_x (m^3)	5.670×10^{-3}	0.05	LN	-	-	-	-
A^c (m^2)	3.335×10^{-2}	0.05	LN	0.05	LN	-	-
I^c_x (m^4)	2.364×10^{-3}	0.05	LN	0.05	LN	0.05	LN
Z^c_x (m^3)	8.374×10^{-3}	0.05	LN	-	-	0.05	LN
F_y (kN/m^2)	2.482×10^5	-	-	-	-	0.1	LN
ξ	0.02	0.15	LN	0.15	LN	0.15	LN
g_e	1.0	0.20	Type I	0.20	Type I	-	-
	2.3	-	-	-	-	0.2	Type I

Numerical Results for Example 1

$$g(\mathbf{X}) = \delta_{allow} - y_{max}(\mathbf{X}) = 2.286 - y_{max}(\mathbf{X})$$

Serviceability Limit State: Case 1 – 9 random variables

MCS	P_f	0.03627 (β=1.7957)			
	CPU Time	83479 sec (over 23 hours)			
Proposed Algorithm	Scheme	Scheme ① (SD)	Scheme ② (CCD)	Scheme ③ (SD w/0 & w)	Scheme ④ (SD w/0 &ccd)
	β	1.771	1.801	1.814	1.800
	P_f	0.03824	0.03583	0.03482	0.03592
	CPU Time	54 sec	1340 sec	84 sec	476 sec
	TNSP	57	1593	93	569
Error		-5.43 %	1.21 %	4.00 %	0.96 %

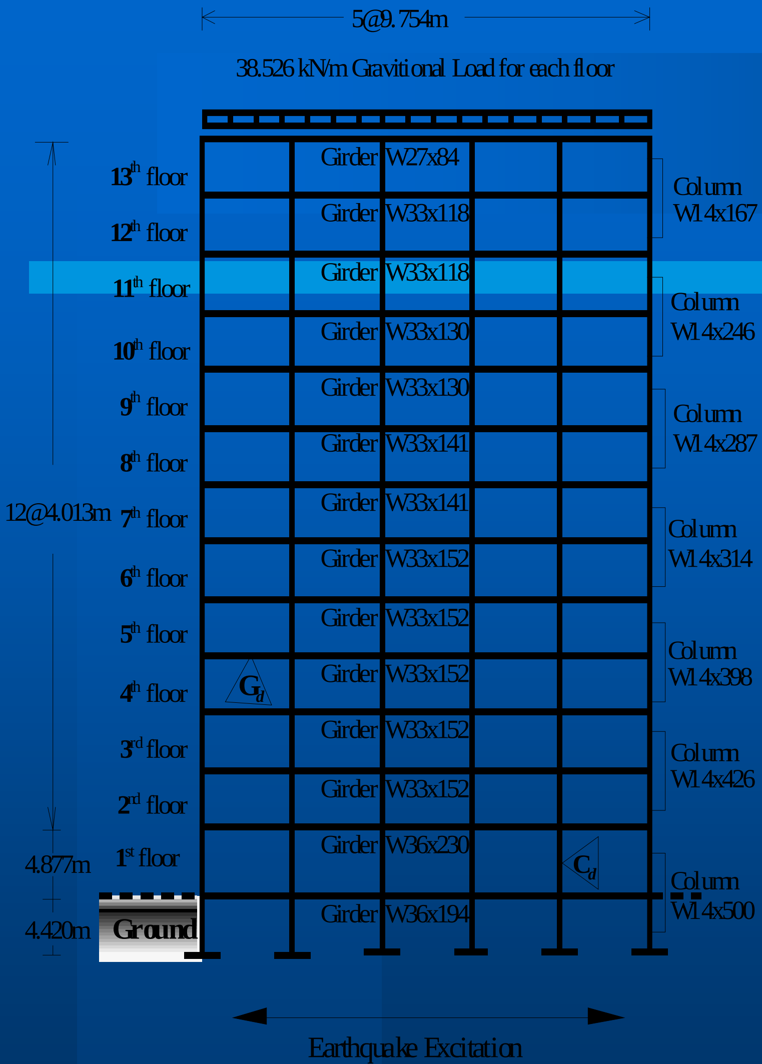
Numerical Results for Example 1

$$g(\mathbf{X}) = \delta_{allow} - y_{max}(\mathbf{X}) = 2.286 - y_{max}(\mathbf{X})$$

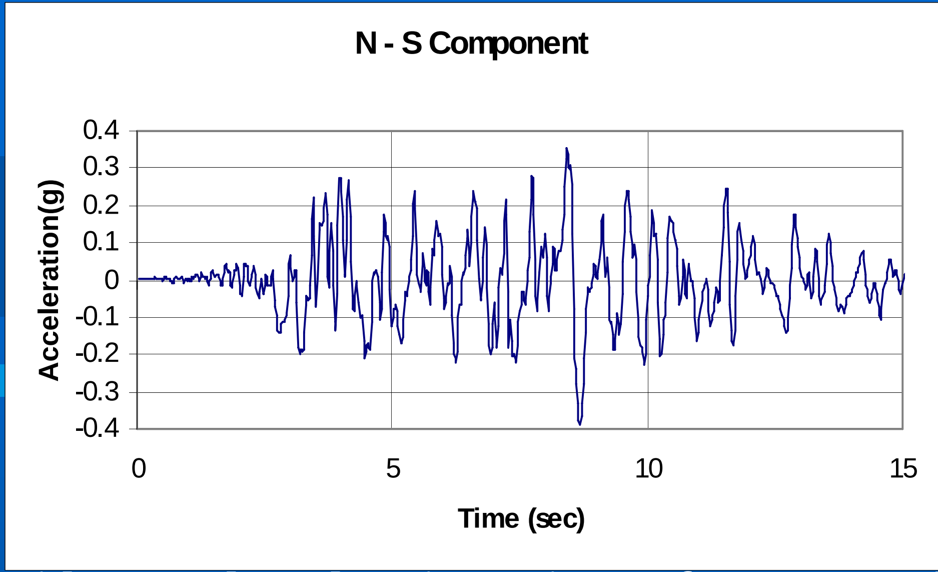
Serviceability Limit State: Case 2 – 7 random variables					
MCS	P_f	0.03606 (β=1.7984)			
	CPU Time	81890 sec (less than 23 hours)			
Proposed Algorithm	Scheme	Scheme ①	Scheme ②	Scheme ③	Scheme ④
	β	1.772	1.799	1.822	1.800
	P_f	0.03816	0.03598	0.03424	0.03592
	CPU Time	43 sec	357 sec	62 sec	148 sec
	TNSP	45 (57)	429 (1593)	66 (93)	173 (560)
Error		5.82 %	-0.22 %	-5.05%	-0.39%

Numerical Results for Example 1

Strength Limit State					
MCS	P_f	0.02143 (β=2.0251)			
	CPU Time	80686 sec			
Proposed Algorithm	Scheme	Scheme ①	Scheme ②	Scheme ③	Scheme ④
	β	1.981	2.014	2.054	2.004
	P_f	0.02379	0.02200	0.02001	0.02254
	CPU Time	53 sec	374 sec	73 sec	162 sec
	TNSP	45	429	66	173
Error		-11.02 %	-2.66 %	6.63 %	-5.16%



Thirteen-Story Steel Frame Structure



Northridge Earthquake Time History for 15 seconds (N-S)

Limit States (1)	Serviceability (2)	Strength Limit State (3)	
	$\delta_{allowable} = h/400$	Column, C_d	Girder, G_d
Reliability Index, β	-7.51	-1.39	-1.08
P_f (P_s)	1.000000 (3.08×10^{-14})	0.917936 (0.082064)	0.858837 (0.141163)

Results of Reliability Analysis for Example 2

Steel Frame without shear walls

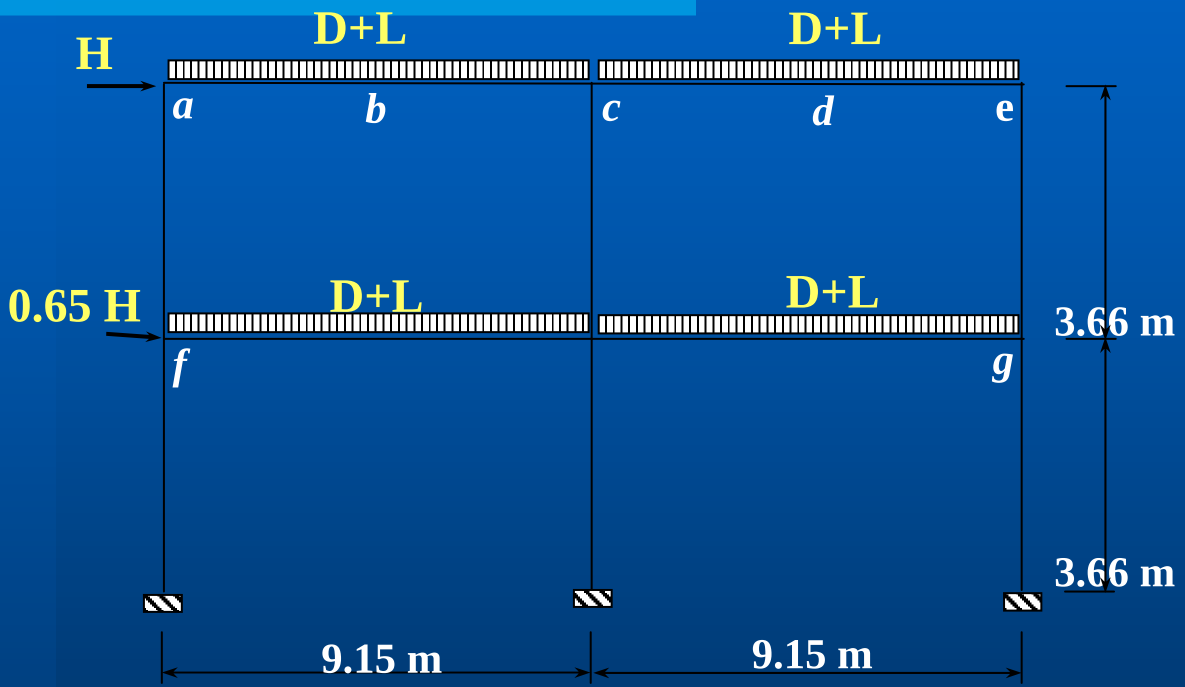
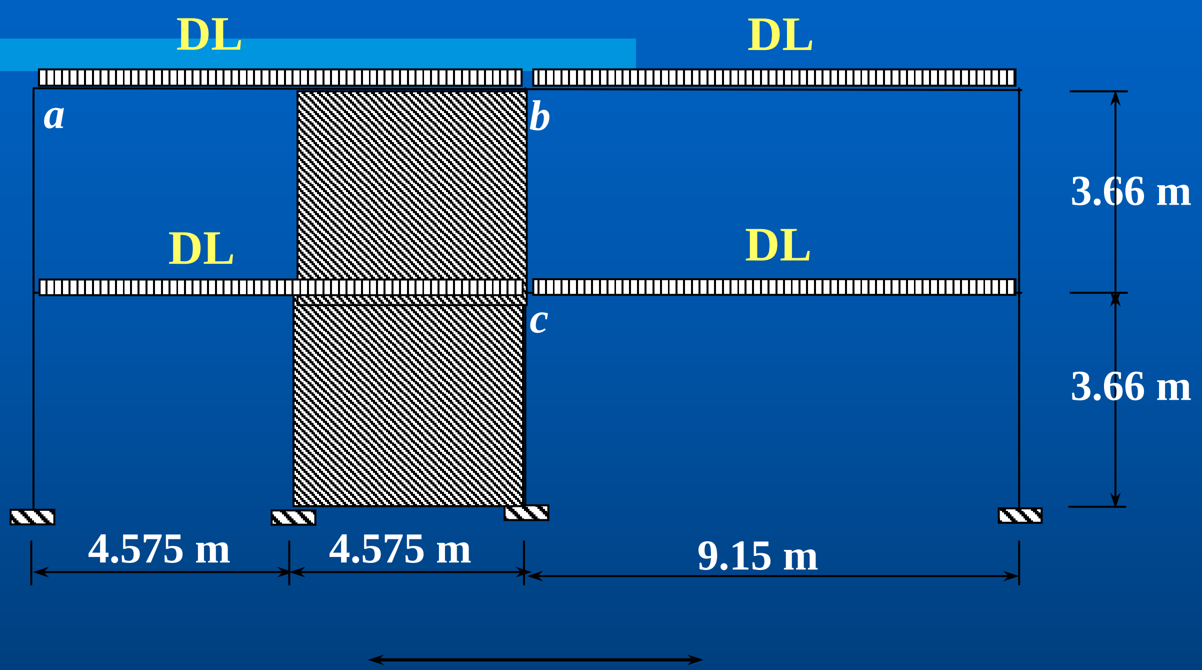


Figure 2. A frame without shear walls

Steel Frame with shear walls



El Centro Earthquake (N-S Component)

(DL=35.04 kN/m)

Figure 1. Numerical model

Material Properties

Frame	Member	Area (cm^2)	I (cm^4)	Section Size
	Beam	113.6	40957	W18x60
	Column	76.1	12903	W12x40
Walls	E_{cr} (Mpa)	ν	Assume $E_c = 0.4E_c$	
	8.55E03	0.17		

TABLE 4. Basic Random Variables

Items	Variables	Nominal Value	Mean/Nominal	C.O.V	Distribution	Comment
Frame	E (Mpa)	2.0E05	1.0	0.06	Log-normal	-
	A^b (cm^2)	113.6	1.0	0.05	Log-normal	Beam W18x60
	I^b (cm^4)	40957	1.0	0.05	Log-normal	
	$Z_x^{\ b}$ (cm^3)	2015	1.0	0.05	Log-normal	
	A^c (cm^2)	76.1	1.0	0.05	Log-normal	Column W12x40
	I^c (cm^4)	12903	1.0	0.05	Log-normal	
	$Z_x^{\ c}$ (cm^3)	942.3	1.0	0.05	Log-normal	
	F_y (Mpa)	248.21	1.05	0.1	Log-normal	
Wall	E_c (Mpa)	2.14E04	1.0	0.18	Log-normal	f_c'=20.68 (Mpa)
	ν	0.17	1.0	0.10	Log-normal	
Dynamic Parameters	ξ	0.02	1.0	0.15	Lognormal	Without shear walls
	g_e	1.0	1.0	0.2	Type I	
	ξ	0.05	1.0	0.15	Lognormal	With shear walls
	g_e	1.0	1.0	0.2	Type I	

b: Beam and *c*: Column
ξ: Damping ratio
g_e: Magnification factor for the amplitude of actual seismic acceleration

Results for a Frame Without and With Shear Walls

	Schemes	RSM		Monte Carlo Simulation	
		P_f	*CPU (s)*	P_f	*CPU (s)*
A frame without shear walls					
Top drift (Node *a*)	Scheme 4	0.9999	134	1.0	98459
A frame with shear walls					
Top drift (Node *a*)	Scheme 4	0.0057	202	0.0049	117832
	Scheme 5	0.0094	295		

Pf = probability of failure, CPU time is for SGI Origin 2000

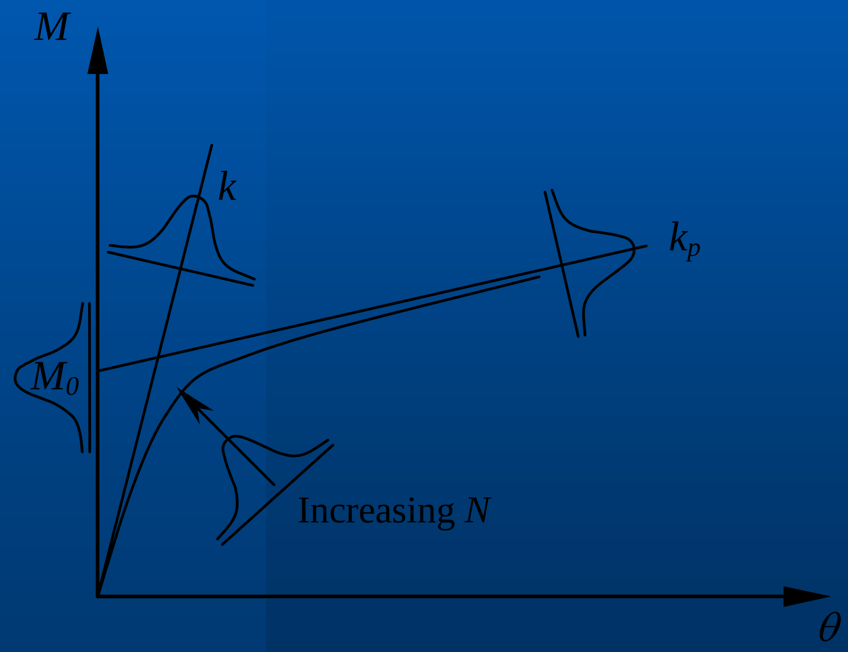
Northridge Earthquake, 1994

- Connections fractured in more than 200 buildings

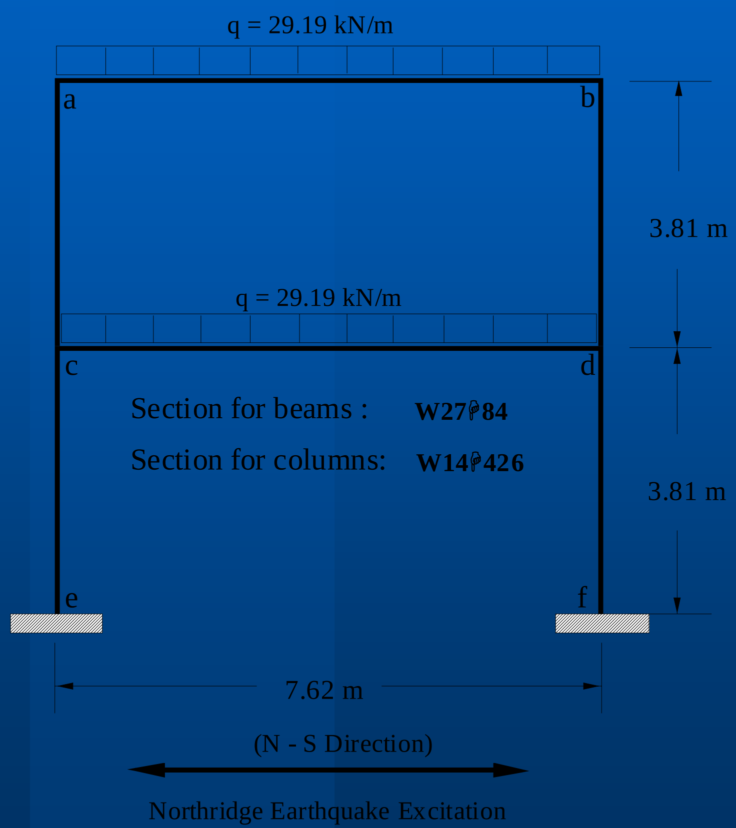
- Connections also fractured in 1989 Loma Prieta earthquake; they went undetected for over 5 years

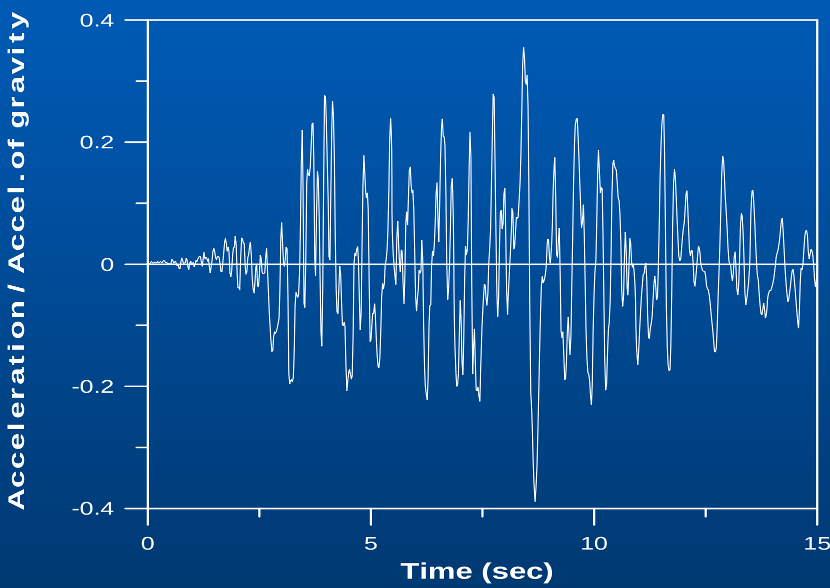
Flexibility of Connections

M-*θ* curves – Loading

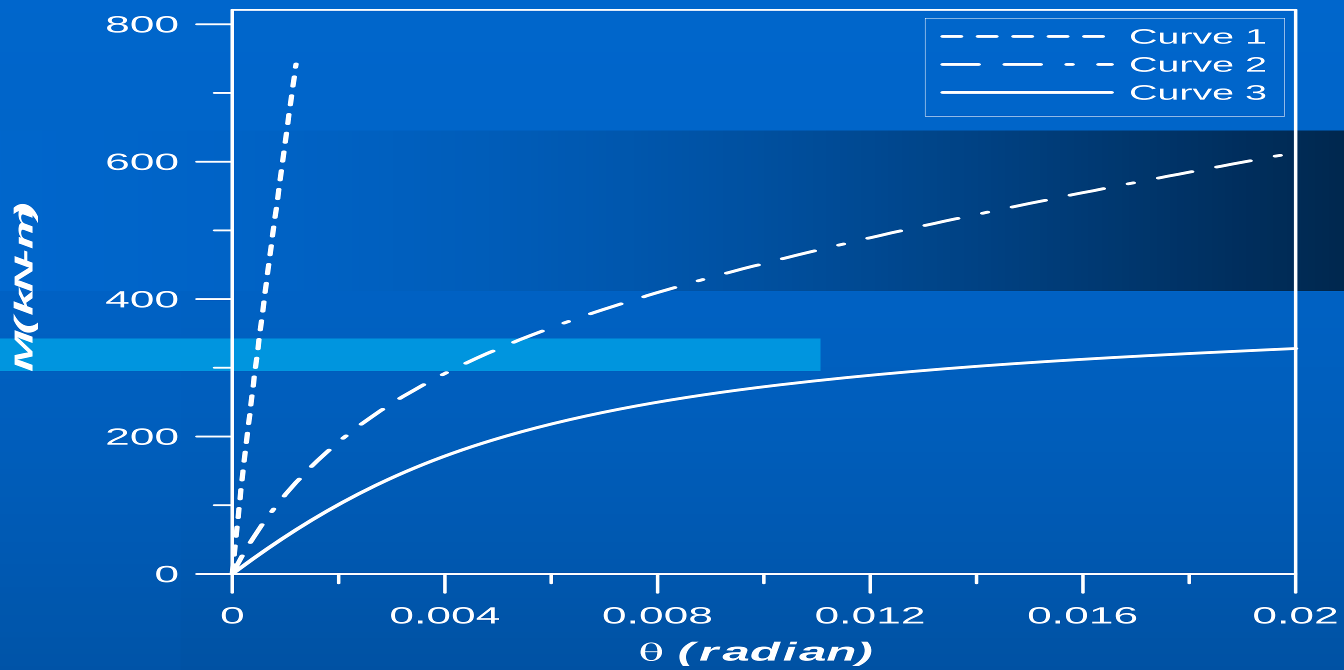


$$M = \frac{(k - k_p)\theta}{\left(1 + \left|\frac{(k - k_p)\theta}{M_0}\right|^N\right)^{1/N}} + k_p\theta$$





Northridge earthquake (N-S) time history



Statistical description of the four parameters in the Richard model

Random Variables	Mean Value			COV	Distribution
	Curve 1	Curve 2	Curve 3		
k (kN·m/rad)	1.13×10^6	1.47×10^5	5.65×10^4	0.15	Normal
k_p (kN·m/rad)	1.13×10^5	1.13×10^4	1.13×10^3	0.15	Normal
M_0 (kN·m)	508.64	452.12	339.09	0.15	Normal
N	0.50	1.00	1.5	0.05	Normal

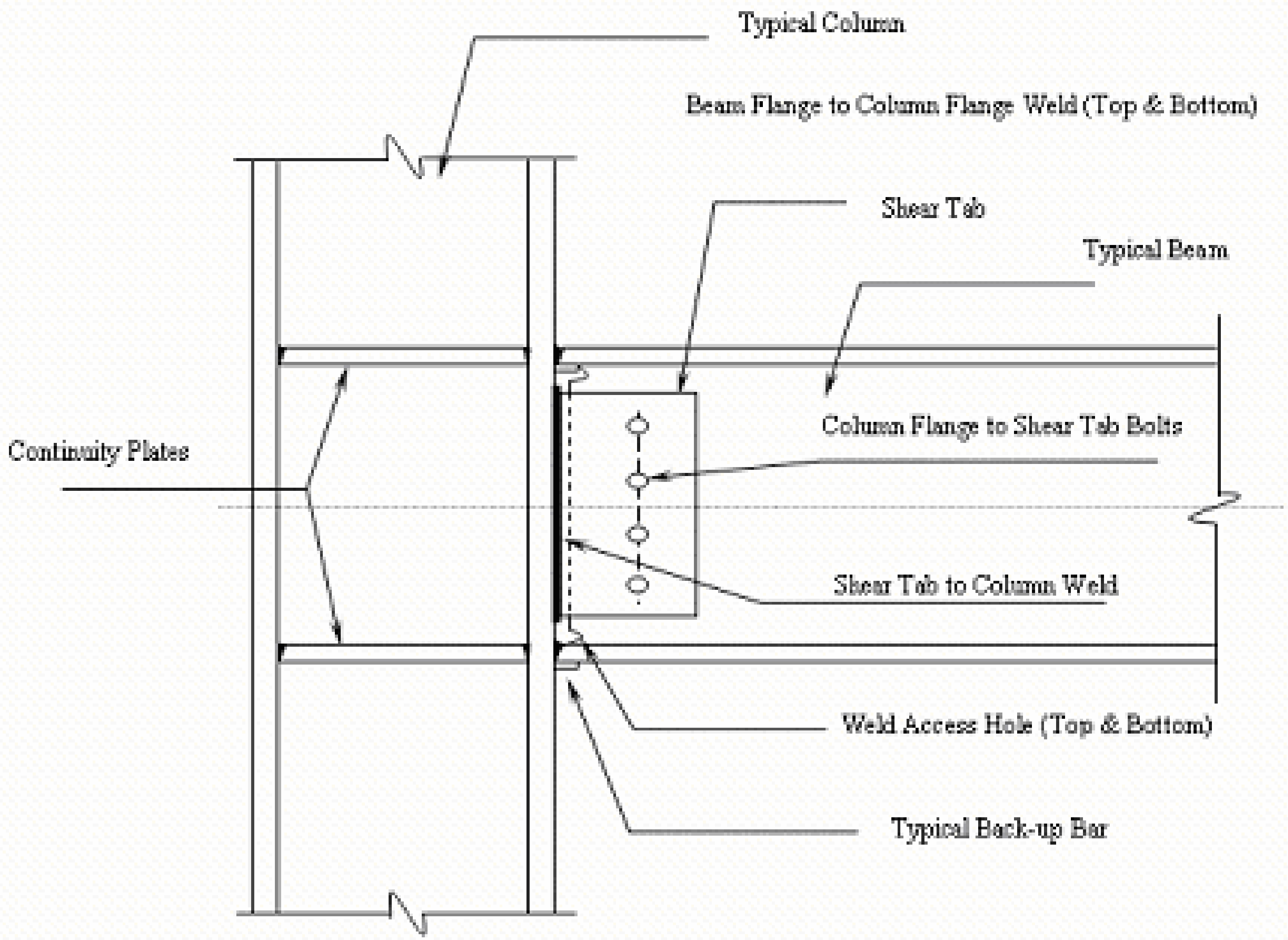
Statistical description of random variables (b: beam, c: column)

Random Variables	Mean Value	Serviceability Limit State		Strength Limit State	
		COV	Dist.	COV	Dist.
E (kN/m^2)	1.9994×10^{8}	0.06	LN	0.06	LN
A^b (m^2)	1.600×10^{-2}	0.05	LN	-	-
I^b_x (m^4)	1.186×10^{-3}	0.05	LN	0.05	LN
Z^b_x (m^3)	3.998×10^{-3}	-	-	0.05	LN
A^c (m^2)	8.065×10^{-2}	0.05	LN	-	-
I^c_x (m^4)	2.747×10^{-3}	0.05	LN	0.05	LN
F_y (kN/m^2)	2.4822×10^{5}	-	-	0.10	LN
ξ	0.05	0.15	LN	0.15	LN
g_e	1.00	0.20	Type I	0.20	Type I

Result of reliability analysis of example

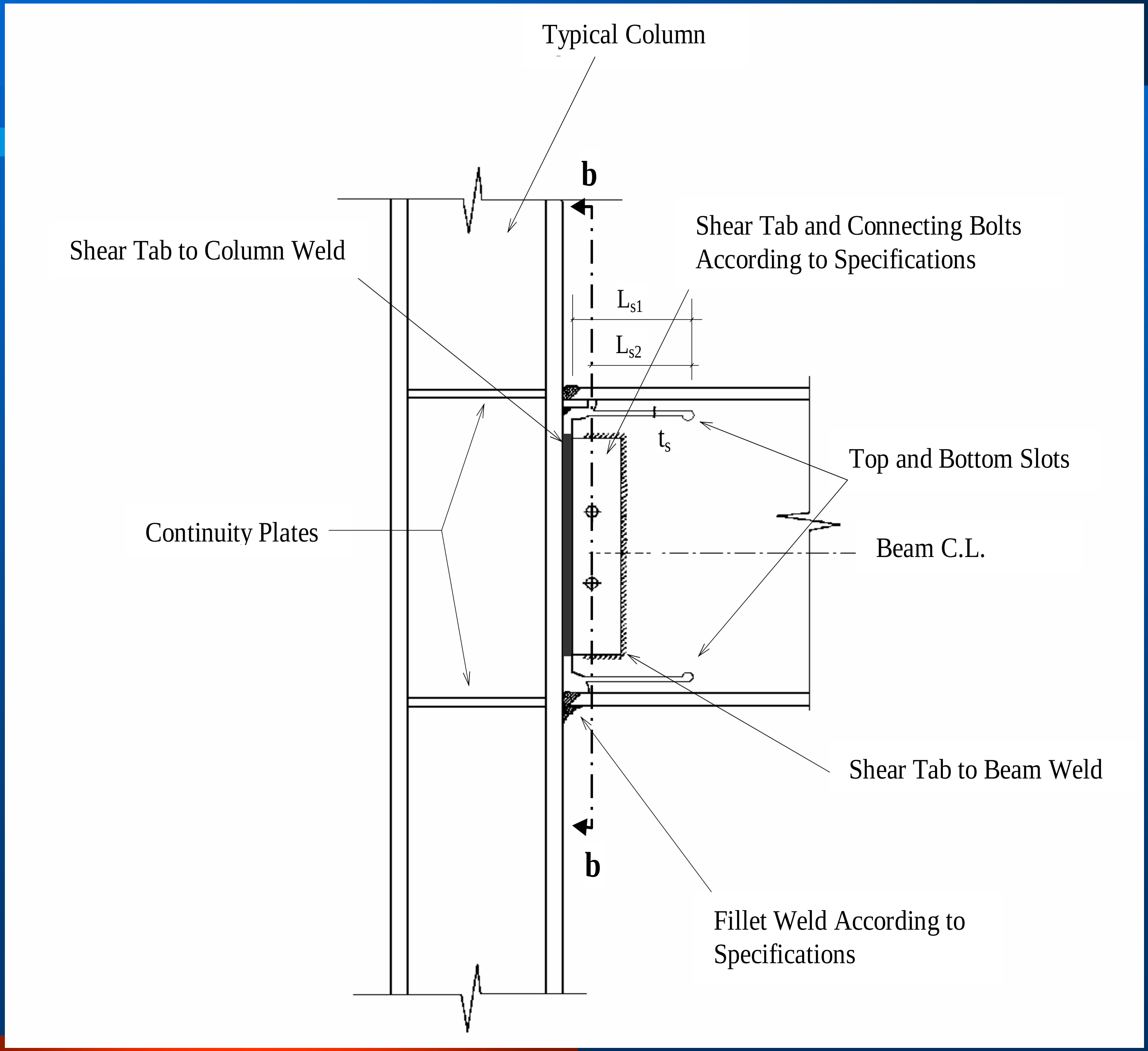
Limit State	FR Connections	PR Connections		
		Curve 1	Curve 2	Curve 3
Serviceability Limit State	$\beta = 1.920$	$\beta_1 = 1.274$	$\beta_2 = -0.008$	$\beta_3 = -0.899$
Strength Limit State	$\beta = 3.944$	$\beta_1 = 2.351$	$\beta_2 = 2.558$	$\beta_3 = 3.156$

Pre-Northridge Connections



Pre-Northridge BWWF Connection Detail

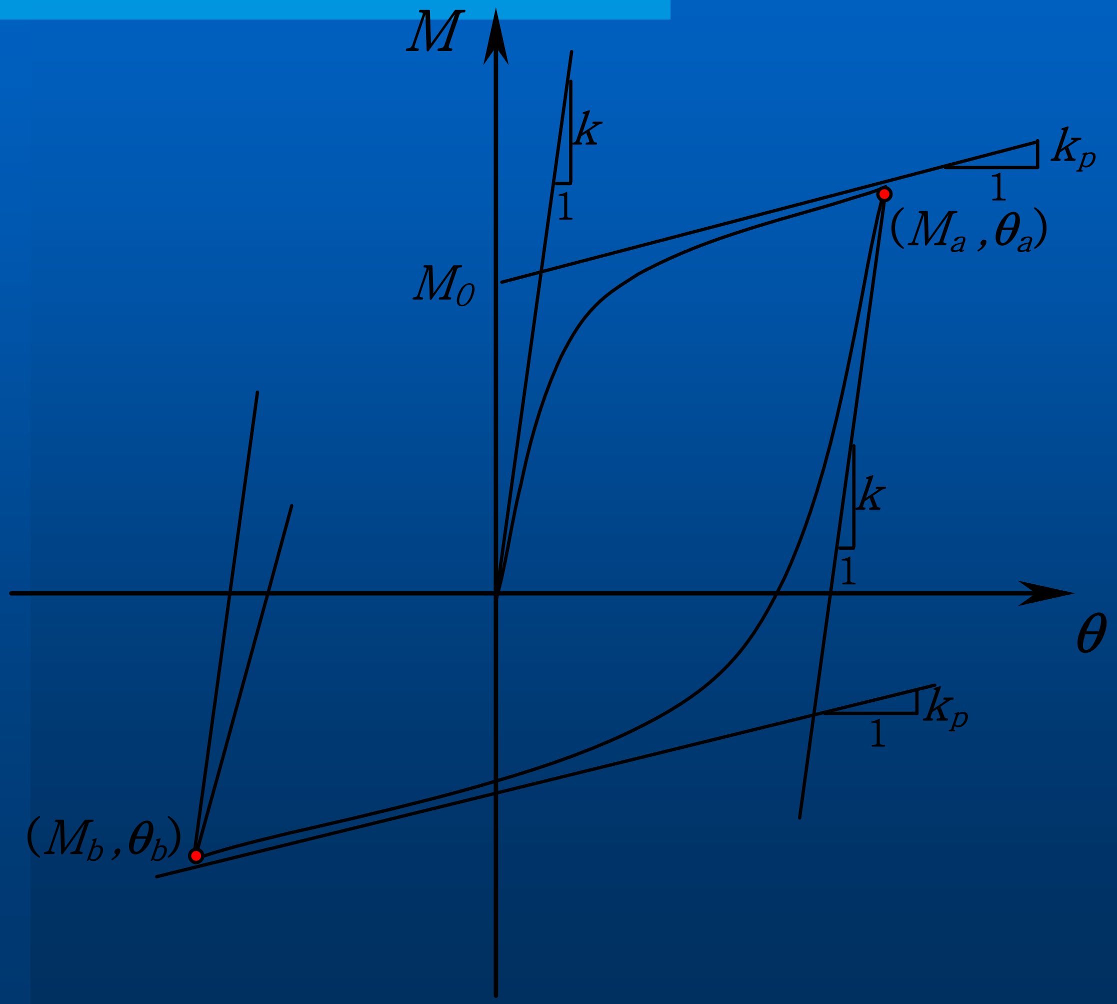
Post-Northridge Connections



Unloading and Reloading

Post-Northridge Connections

Masing Rule



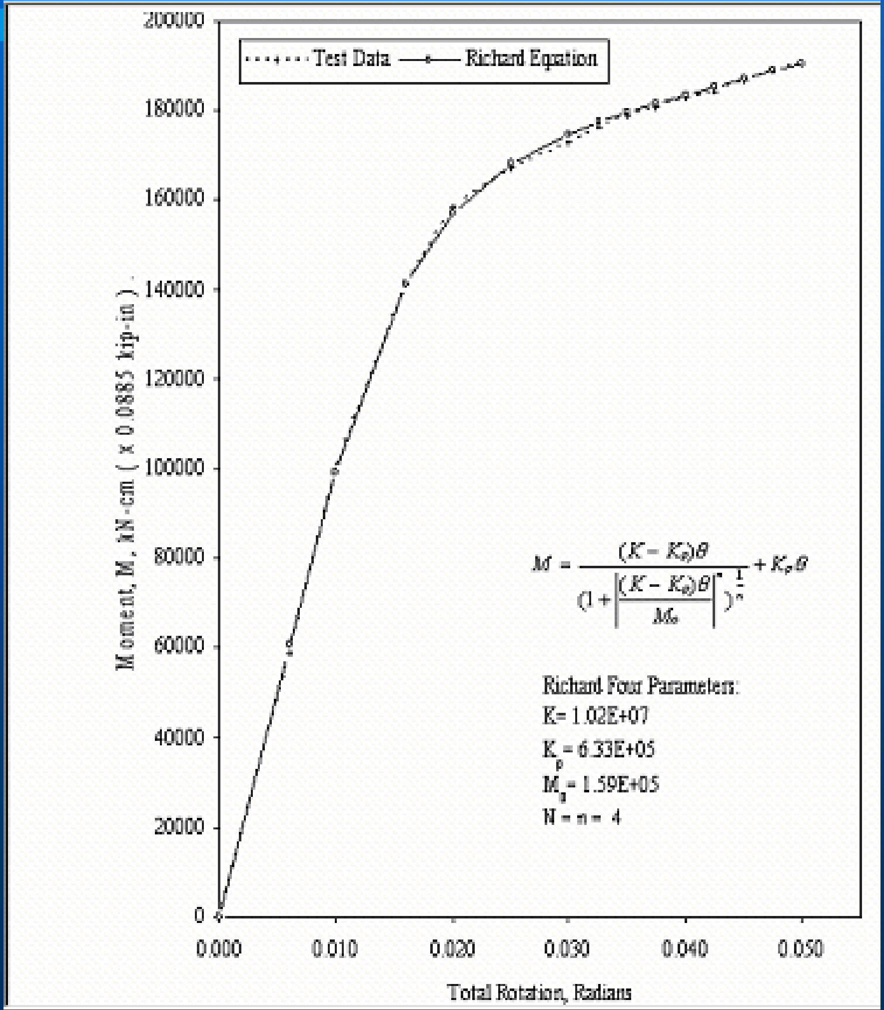
Test No.	Beam	Column	I_b (cm^4)	I_c (cm^4)	L_b (cm)	L_c (cm)
18	W33×141	W14×283	310092	159833	439	391
20	W27×94	W14×176	136108	89074	457	356
22	W36×300	W14×311	844950	180228	384	391
24	W24×94	W30×132	112382	240166	381	396
26	W36×170	W30×235	437043	486991	409	391

b : beam, c : column

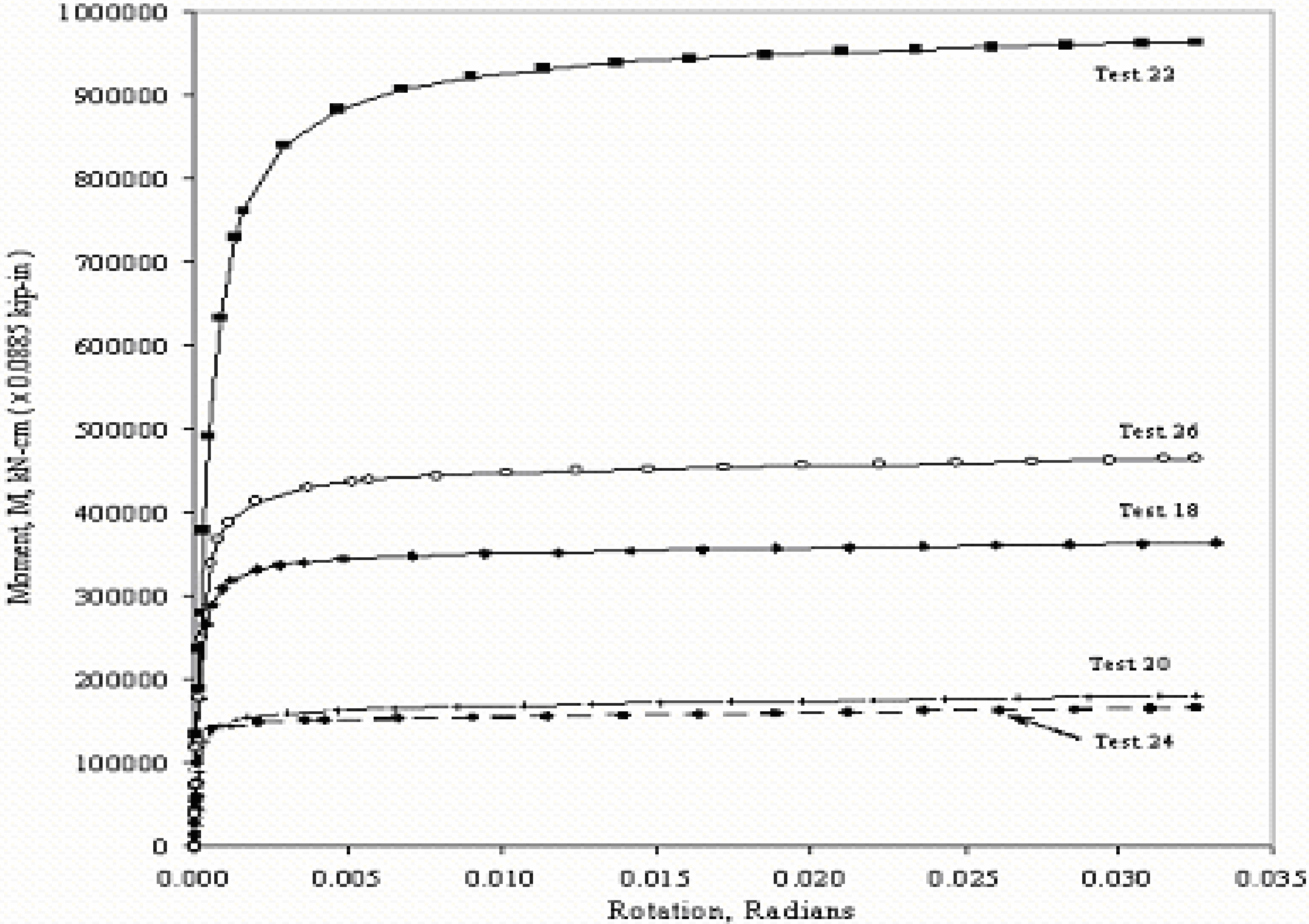
Table 2. Parameters of Richard Equation for M-θ Curves

Test No.	Connection Parameters			
	K^1	$K_p{}^1$	$M_0{}^2$	N
18	2.938E+09	4.519E+05	3.503E+05	1.0
20	2.712E+09	4.519E +05	1.672E+05	0.8
22	2.260E+09	4.519E +05	9.491E+05	1.0
24	3.390E+09	4.519E +05	1.509E+05	1.0
26	3.955E+09	4.519E +05	4.519E+05	0.8

Note: [1]kN·cm/rad, [2]kN·cm

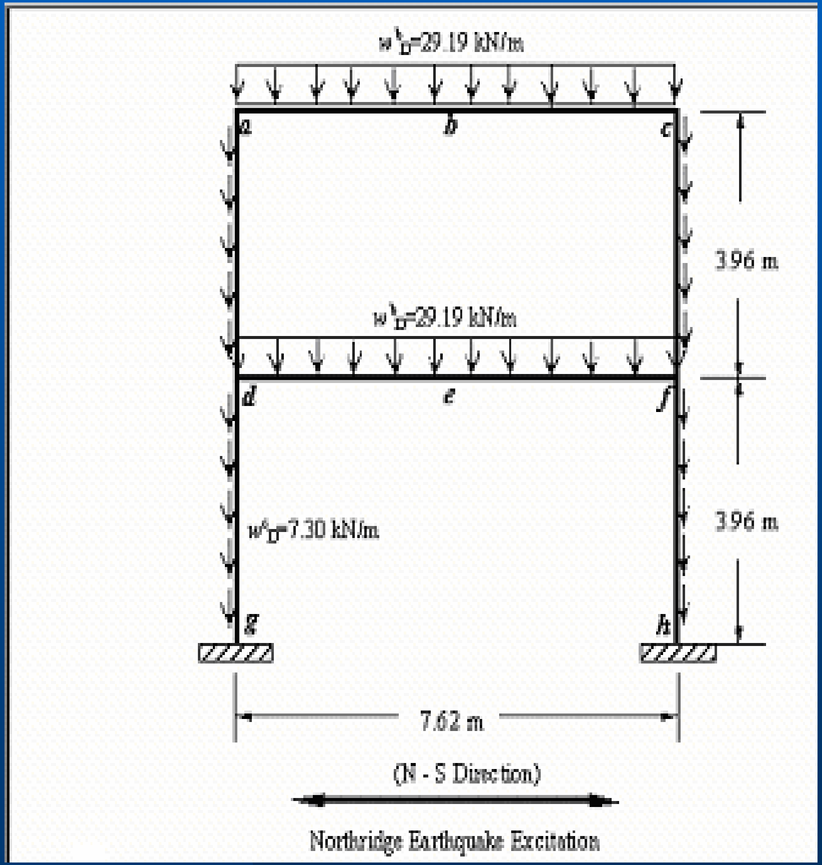


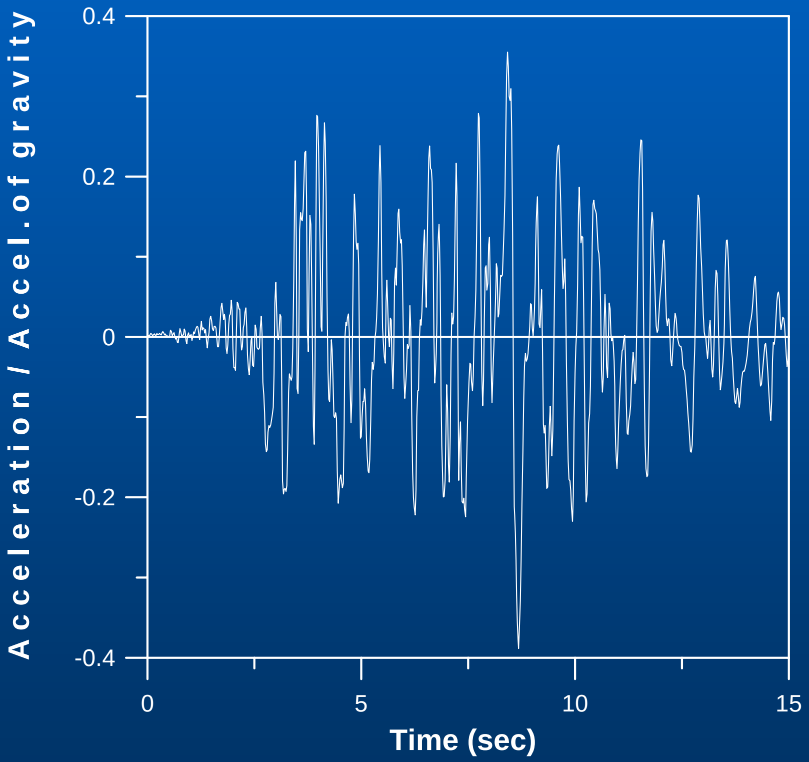
Test Data
Richard Equation
Moment, M, kN-cm (x 0.0885 kip-in)
Total Rotation, Radians
200000
180000
160000
140000
120000
100000
80000
60000
40000
20000
0
0.000
0.010
0.020
0.030
0.040
0.050
$M = \frac{(K - K_p)\theta}{\left(1 + \left|\frac{(K - K_p)\theta}{M_o}\right|^n\right)^{\frac{1}{n}}} + K_p\theta$
Richard Four Parameters:
K= 1.02E+07
K_p = 6.33E+05
M_o= 1.59E+05
N = n = 4



1000000
900000
800000
700000
600000
500000
400000
300000
200000
100000
0
0.000
0.005
0.010
0.015
0.020
0.025
0.030
0.035
Rotation, Radians
Moment, M, kN-cm (x 0.0885 kip-in)
Test 22
Test 26
Test 18
Test 20
Test 24

Two-story Steel Frame Structure and Earthquake Time History





. Statistical Description of Random Variables (b: beam, c: column)

Random Variable	Mean Value					Serviceability		Strength Limit State			
	Test 18	Test 20	Test 22	Test 24	Test 26	Node at *c*		Beam (*e-f*)		Column (*f-h*)	
						COV	Dist.	COV	Dist.	COV	Dist.
E (kN/m^2)	1.999×10^{8}	1.999×10^{8}	1.999×10^{8}	1.999×10^{8}	1.999×10^{8}	0.06	LN	0.06	LN	0.06	LN
A^b (m^2)	2.684×10^{-2}	1.787×10^{-2}	5.697×10^{-2}	1.787×10^{-2}	3.226×10^{-2}	0.05	LN	-		-	-
I^b_x (m^4)	3.101×10^{-3}	1.361×10^{-3}	8.449×10^{-3}	1.124×10^{-3}	4.370×10^{-3}	0.05	LN	0.05	LN	0.05	LN
Z^b_x (m^3)	8.423×10^{-3}	4.556×10^{-3}	2.065×10^{-2}	4.162×10^{-3}	1.095×10^{-2}	-	-	0.05	LN	-	-
A^c (m^2)	5.374×10^{-2}	3.342×10^{-2}	5.897×10^{-2}	2.510×10^{-2}	4.452×10^{-2}	0.05	LN	-		-	-
I^c_x (m^4)	1.598×10^{-3}	8.907×10^{-3}	1.802×10^{-3}	2.402×10^{-3}	4.870×10^{-3}	0.05	LN	0.05	LN	0.05	LN
Z^c_x (m^3)	8.882×10^{-3}	5.244×10^{-3}	9.881×10^{-3}	7.161×10^{-3}	1.385×10^{-2}	-	-	-	-	0.05	LN
F_y (kN/m^2)	3.447×10^{5}	3.447×10^{5}	3.447×10^{5}	3.447×10^{5}	3.447×10^{5}	-	-	0.10	LN	0.10	LN
ξ	0.05	0.05	0.05	0.05	0.05	0.15	LN	0.15	LN	0.15	LN
g_e	2.00	1.50	2.00	1.50	2.00	0.20	Type I	0.20	Type I	0.20	Type I

Statistical Description of the Four Parameters in the Richard Model

Random variables	Value Mean					COV	Dist.
	Test 18	Test 20	Test 22	Test 24	Test 26		
K (kN.cm/rad)	2.938×10^9	2.712×10^9	2.260×10^9	3.390×10^9	3.955×10^9	0.15	Normal
k_0 (kN.cm/rad)	4.519×10^3	4.519×10^3	4.519×10^3	4.519×10^3	4.519×10^3	0.15	Normal
M_0 (kN.cm)	3.503×10^3	1.672×10^3	9.491×10^3	1509×10^3	4.519×10^3	0.15	Normal
N	1.0	0.8	1.0	1.0	0.8	0.05	Normal

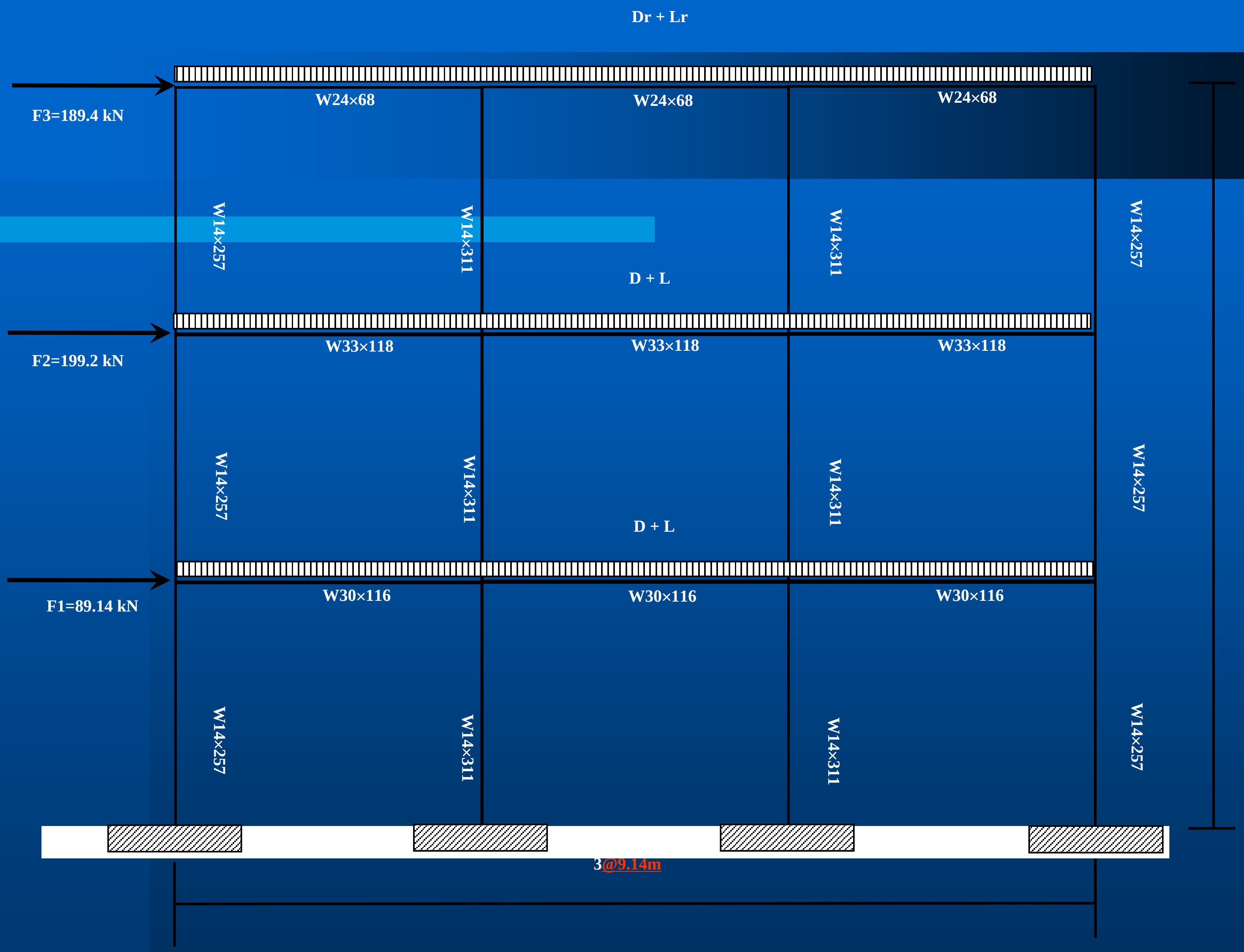
Reliability Analysis Result for Test #24

Limit State		Serviceability		Strength Limit State			
		Node at *c*		Beam (*e*-*f*)		Column (*f*-*h*)	
Case		FR	PR	FR	PR	FR	PR
MCS	P_f	0.87525	0.87820	0.00040	0.00025	0.00130	0.00105
	$\beta \approx \Phi^{-1}(1-P_f)$	-1.152	-1.166	3.353	3.481	3.011	3.076
	NOS*1	20,000	20,000	20,000	20,000	20,000	20,000
Proposed Algorithm	No. of RV	7	11	7	11	7	11
	Scheme	2	1	2	1	2	1
	P_f	0.87055	0.90303	0.00045	0.00018	0.00125	0.00096
	β	-1.129	-1.299	3.323	3.574	3.024	3.101
	Error	0.5 %	-2.8 %	0.9 %	-2.7 %	-0.4 %	-0.8 %
	TNSP*2	173	124	173	124	173	124

*1 number of simulation for deterministic FEM analyses
*2 total number of sampling points (total number of deterministic FEM analyses)

Reliability Indexes for all five tests

Test No.	Serviceability		Strength Limit State			
	Node at *c*		Beam (*e*-*f*)		Column (*f*-*h*)	
	BWWF (FR)	BWWF-AD (PR)	BWWF (FR)	BWWF-AD (PR)	BWWF (FR)	BWWF-AD (PR)
# 18	-0.972	-1.013	3.613	3.746	3.795	3.660
# 20	-1.970	-1.989	2.994	3.021	3.066	3.053
# 22	1.175	1.033	7.267	7.548	4.719	4.659
# 24	-1.129	-1.299	3.323	3.574	3.024	3.001
# 26	1.642	1.598	5.547	5.743	4.804	4.714



Dr + Lr
F3=189.4 kN
W24×68
W24×68
W24×68
W14×257
W14×311
W14×311
W14×257
D + L
F2=199.2 kN
W33×118
W33×118
W33×118
W14×257
W14×311
W14×311
W14×257
D + L
F1=89.14 kN
W30×116
W30×116
W30×116
W14×257
W14×311
W14×311
W14×257
3@3.96m
3@9.14m

Table 1: Member sizes for the steel frame

Story	Columns		Beam
	Exterior	Interior	
0 – 1	W14×257	W14×311	W30×116
1 – 2	W14×257	W14×311	W33×118
2 – 3	W14×257	W14×311	W24×68

Statistical Description of Random Variables

Item		Random Variable	Mean Value	COV	Dist.
Member	All	E (kN/m^2)	1.999×10^{8}	0.06	LN
	W24×68	A (m^2)	1.297×10^{-2}	0.05	LN
		I_x (m^4)	7.617×10^{-4}	0.05	LN
	W33×118	A (m^2)	2.239×10^{-2}	0.05	LN
		I_x (m^4)	2.456×10^{-3}	0.05	LN
	W30×116	A (m^2)	2.206×10^{-2}	0.05	LN
		I_x (m^4)	2.052×10^{-3}	0.05	LN
	W14×257	A (m^2)	4.877×10^{-2}	0.05	LN
		I_x (m^4)	1.415×10^{-3}	0.05	LN
	W14×311	A (m^2)	5.897×10^{-2}	0.05	LN
		I_x (m^4)	1.802×10^{-3}	0.05	LN

Load	Gravitational Load	Dr (kN/m)	30.65	0.10	LN
		D (kN/m)	35.02		
		Lr (kN/m)	5.25	0.25	Type I
		L (kN/m)	25.39		
	Seismic Load	F1 (kN)	89.14	0.37	Type I
		F2 (kN)	199.2		
		F3 (kN)	189.4		

LN = lognormal distribution

Reliability Indexes for the Serviceability Limit

Item	Connection Type		
	FR	BWWF-AD	BWWF
Reliability Index	2.602	2.605	1.585
Probability of failure	4.634×10^{-3}	4.594×10^{-3}	56.48×10^{-3}
No. of R.V.	16	28	28
Sensitivity indexes of 5 most sensitive random variables	F3 0.814	F3 0.769	F3 0.764
	F2 0.532	F2 0.445	F2 0.433
	E -0.173	E -0.215	k^1 (Assembly C) -0.207
	Ix (W14×311) -0.102	k^1 (Assembly C) -0.165	M_0 (Assembly C) -0.186
	Ix (W33×118) -0.086	k^1 (Assembly B) -0.135	k^1 (Assembly B) -0.164

Parameters of Richard Equation for M-θ Curves

Connection Assembly				Connection Parameters			
ID.		Beam	Column	k^1	$k_p{}^1$	$M_0{}^2$	N
BWWF	A	W24×68 W24×68	W14×257 W14×311	2.51×10^7	5.56×10^5	4.16×10^4	1.1
	B	W30×116 W30×116	W14×257 W14×311	3.95×10^7	9.19×10^5	5.65×10^4	1.1
	C	W33×118 W33×118	W14×257 W14×311	5.08×10^7	1.14×10^5	6.79×10^4	1.1

Parameters of Richard Equation for M-θ Curves – Cont.

BWWF-AD	A	W24×68 W24×68	W14×257 W14×311	1.00×10^{9}	4.52×10^{5}	9.64×10^{4}	1.0
	B	W30×116 W30×116	W14×257 W14×311	2.14×10^{9}	4.52×10^{5}	2.21×10^{5}	1.0
	C	W33×118 W33×118	W14×257 W14×311	2.34×10^{9}	4.52×10^{5}	2.44×10^{5}	1.0

Note: [1]kN·cm/rad, [2]kN·cm

Statistical Description of the Four Parameters in the Richard Model

	R.V.	Mean Value			COV	Dist.
		Assembly A	Assembly B	Assembly C		
BWWF	K[1]	2.51×10^7	3.95×10^7	5.08×10^7	0.15	N
	k_p[1]	5.56×10^5	9.19×10^5	1.14×10^6	0.15	N
	M_0[2]	4.16×10^4	5.65×10^4	6.79×10^4	0.15	N
	N	1.1	1.1	1.1	0.05	N
BWWF-AD	K[1]	1.00×10^9	2.14×10^9	2.34×10^9	0.15	N
	k_p[1]	4.52×10^5	4.52×10^5	4.52×10^5	0.15	N
	M_0[2]	9.64×10^4	2.21×10^5	2.44×10^5	0.15	N
	N	1.0	1.0	1.0	0.05	N

Note: [1]kN·cm/rad, [2]kN·cm, N = normal distribution

Figure 5: A three-story three-bay frame structure with two shear walls in the center of the 1st and 2nd floors

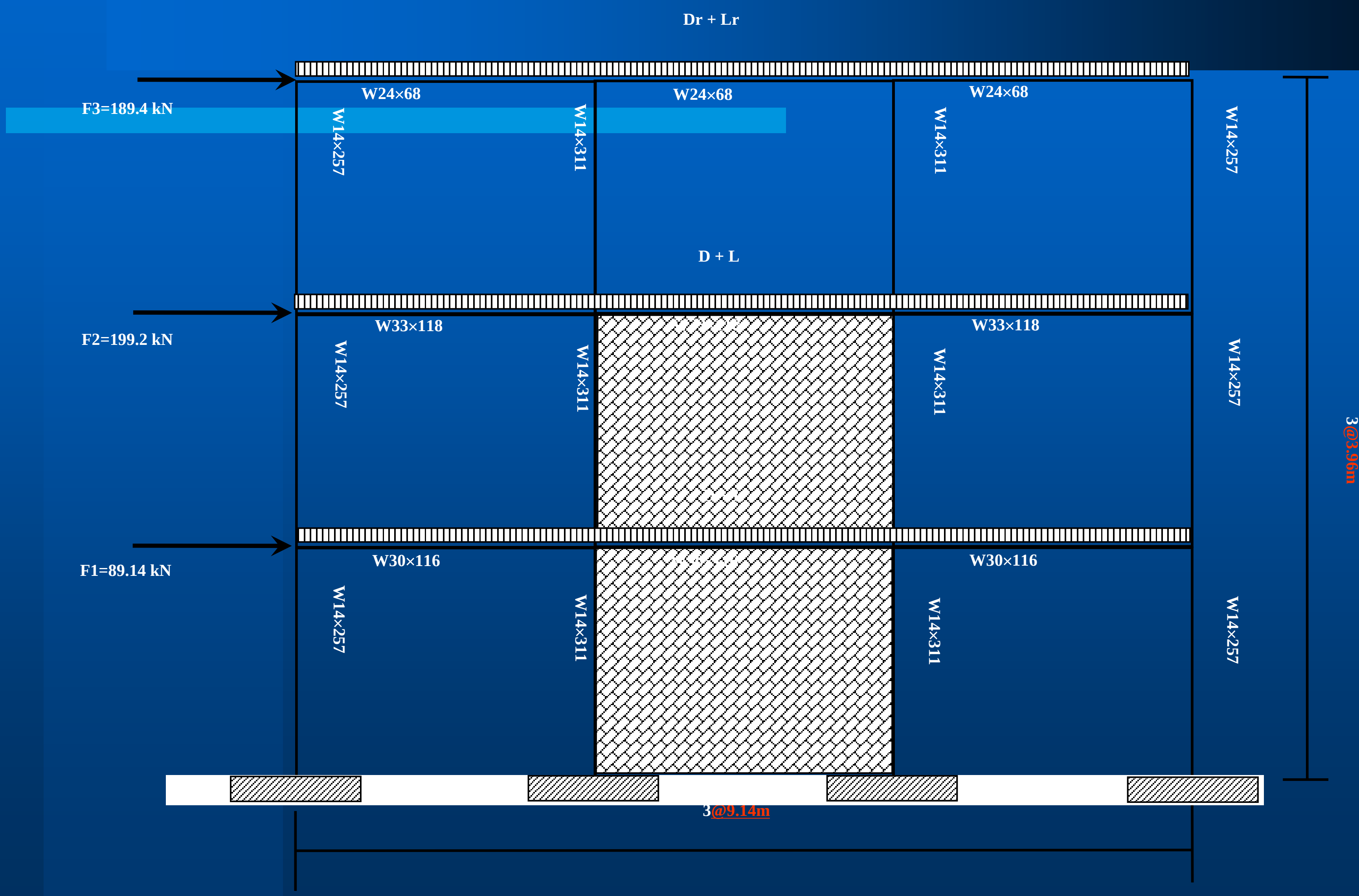
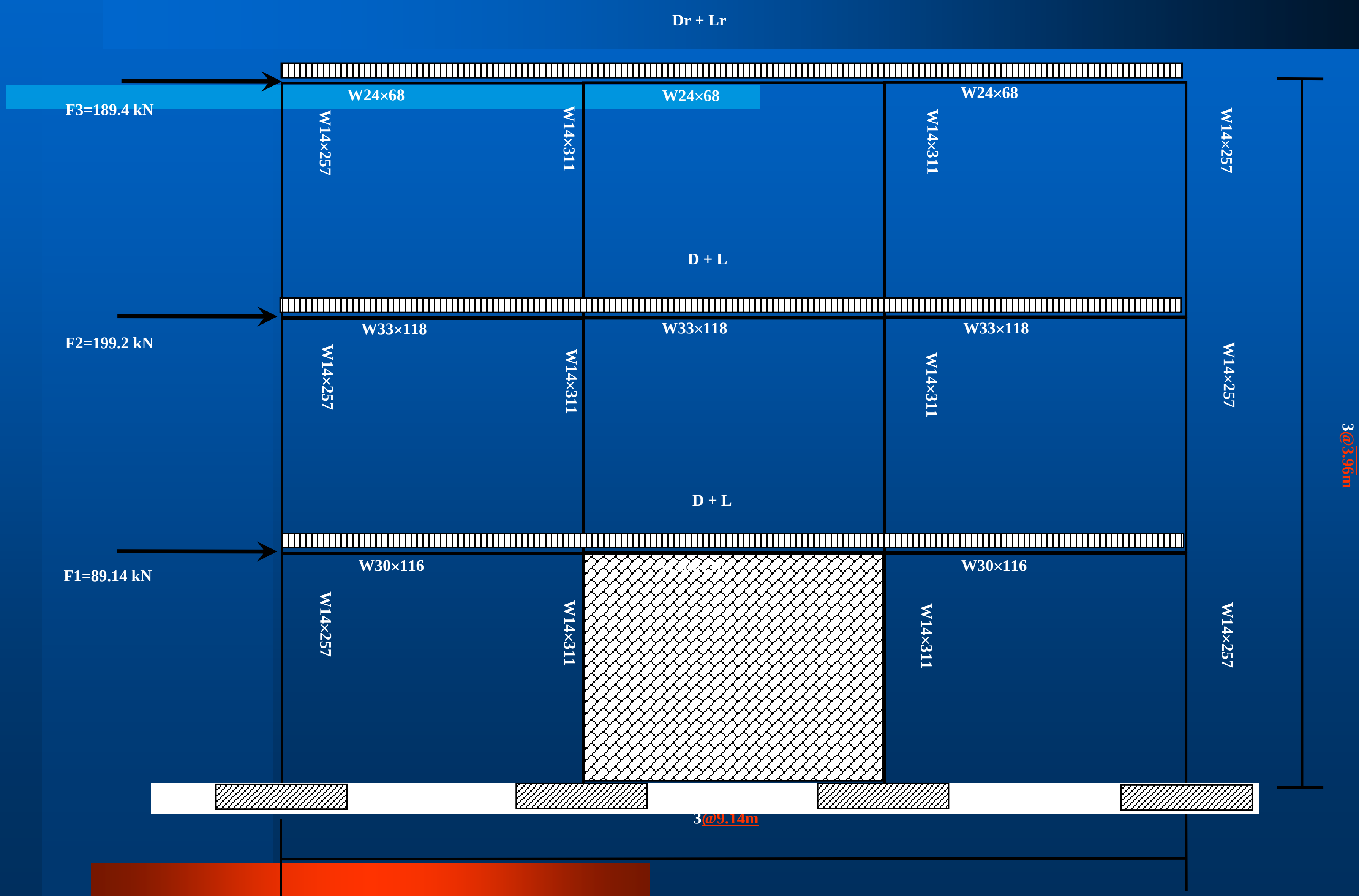


Figure 6: A three-story three-bay frame structure with only one shear wall in the center of the 1st floor



Statistical description of random variables for the shear walls

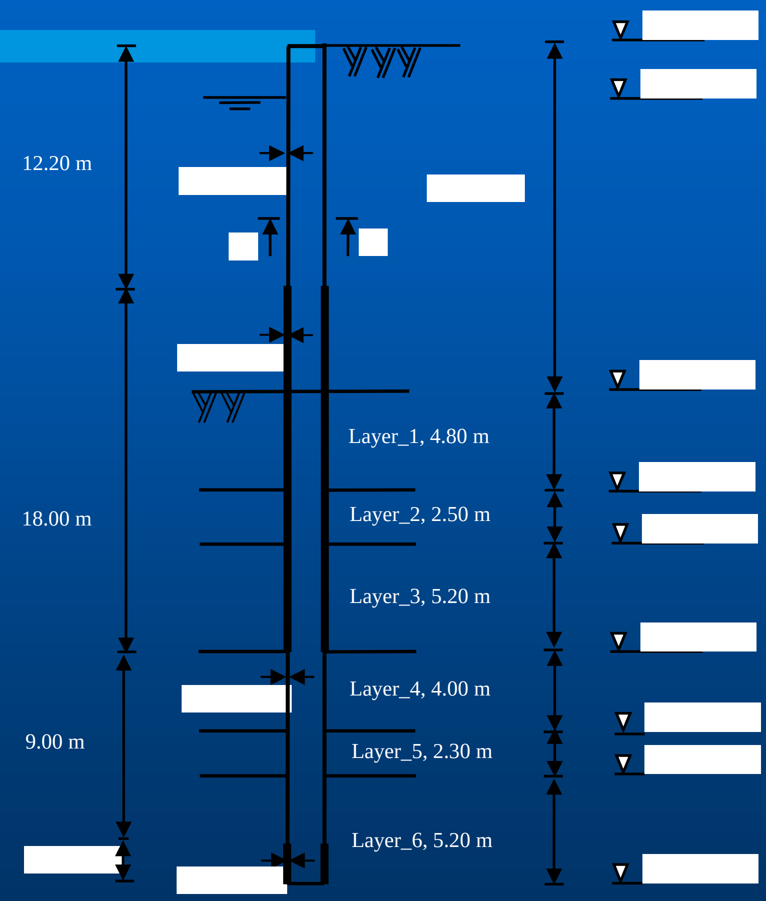
Item	Random Variables	Mean Value	COV	Distribution
Shear Wall	E_C (kN/m^2)	2.137×10^7	0.18	LN
	ν	0.17	0.10	LN

$f_C' = 2.068 \times 10^4$ (kN/m^2)

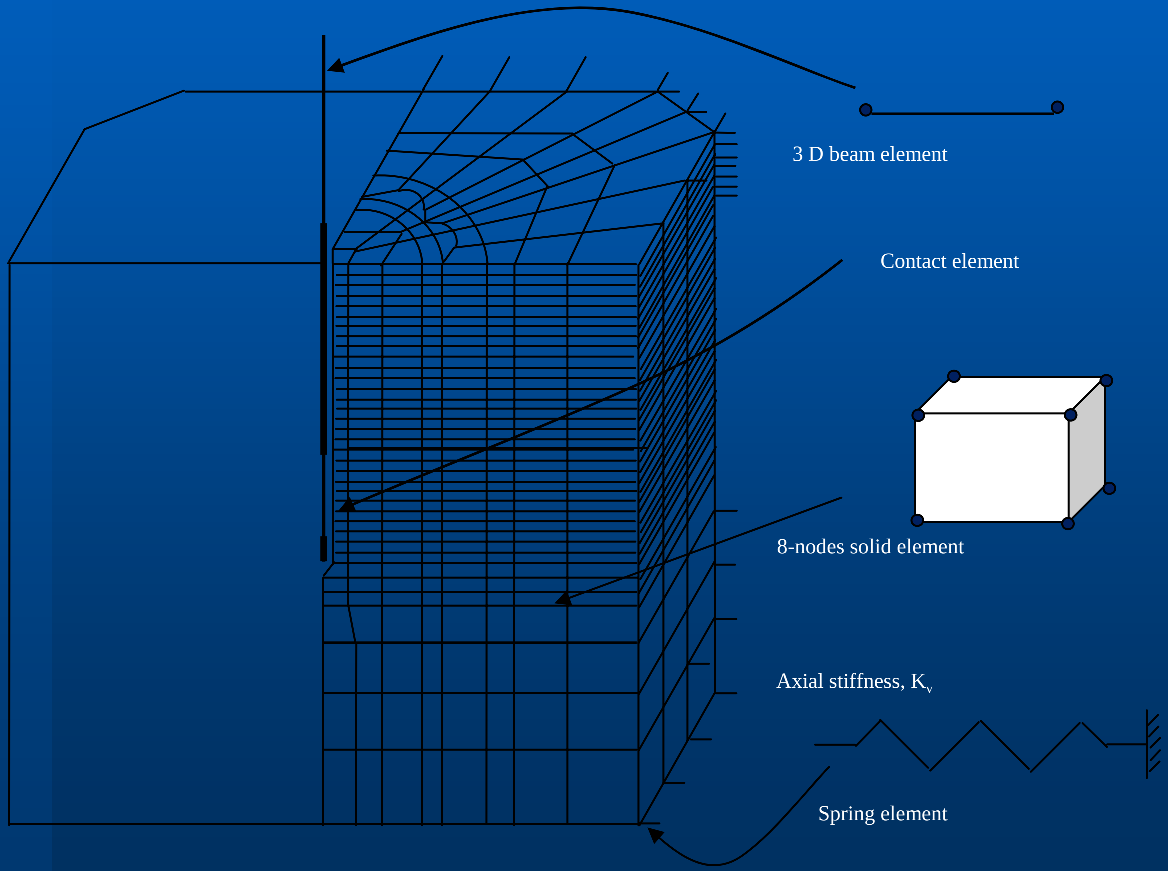
Reliability Indexes for the Serviceability Limit

Item			Connection Type		
			FR	BWWF-AD	BWWF
Without Shear Walls	β		2.602	2.605	1.585
	p_f		4.63×10^{-3}	4.59×10^{-3}	56.48×10^{-3}
With Shear Walls	Fig. 5	β	4.675	4.643	4.396
		p_f	1.479×10^{-6}	1.72×10^{-6}	5.51×10^{-6}
	Fig. 6	β	4.102	4.107	2.823
		p_f	2.05×10^{-5}	2.00×10^{-5}	2.38×10^{-3}

Layout of the pile-soil system



Multi-discretization Finite element model of Pile-Soil system



Conclusions

- A robust structural reliability evaluation procedure is presented
- It can consider all major sources of nonlinearity and uncertainty
- For nonlinear seismic analysis, serviceability limit states could be more critical than commonly used strength limit states
- SFEM procedure provides an alternative to deterministic finite element procedure
- An alternative to random vibration approach

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