

Linear Programming

Graphical method

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LP_2: Graphical Method

Objectives

- To visualize the optimization procedure explicitly
- To understand the different terminologies associated with the solution of LPP
- To discuss an example with two decision variables

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Example

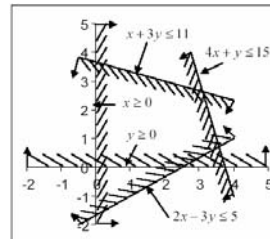
$$\begin{aligned} \text{Maximize } & Z = 6x + 5y \\ \text{subject to } & 2x - 3y \leq 5 && \text{(c-1)} \\ & x + 3y \leq 11 && \text{(c-2)} \\ & 4x + y \leq 15 && \text{(c-3)} \\ & x, y \geq 0 && \text{(c-4 \& c-5)} \end{aligned}$$

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Graphical method: Step - 1



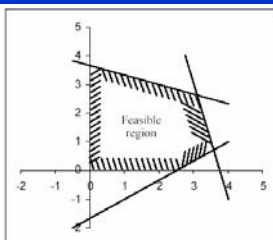
Plot all the constraints one by one on a graph paper

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Graphical method: Step - 2



Identify the common region of all the constraints.

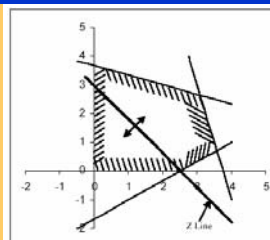
This is known as 'feasible region'

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Graphical method: Step - 3



Plot the objective function, Z, assuming any constant, k, i.e.

$$6x + 5y = k$$

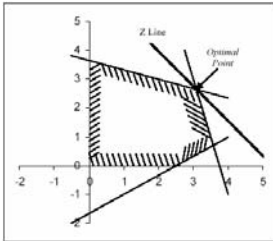
This is known as 'Z line', which can be shifted perpendicularly by changing the value of k.

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Graphical method: Step - 4



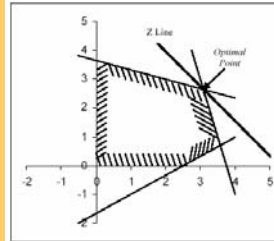
Notice that value of the objective function will be maximum when it passes through the intersection of $x + 3y = 11$ and $4x + y = 15$ (straight lines associated with 2nd and 3rd constraints). This is known as 'Optimal Point'

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Graphical method: Step - 5



Thus the *optimal point* of the present problem is

$$x^* = 3.091$$

$$y^* = 2.636$$

And the optimal solution is

$$6x^* + 5y^* = 31.726$$

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Different cases of optimal solution

A linear programming problem may have

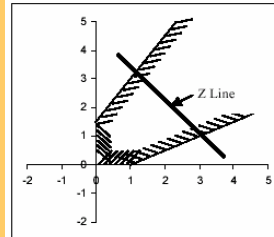
1. A unique, finite solution (example already discussed)
2. An unbounded solution,
3. Multiple (or infinite) number of optimal solution,
4. Infeasible solution, and
5. A unique feasible point.

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Unbounded solution: Graphical representation



Situation: If the feasible region is not bounded

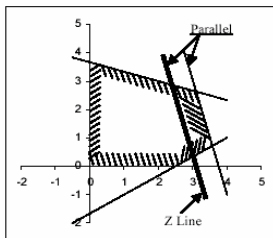
Solution: It is possible that the value of the objective function goes on increasing without leaving the feasible region, i.e., unbounded solution

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Multiple solutions: Graphical representation



Situation: *Z line* is parallel to any side of the feasible region

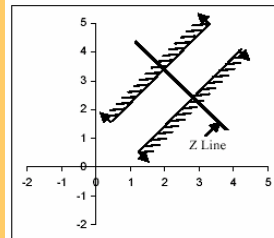
Solution: All the points lying on that side of the feasible region constitute optimal solutions

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Infeasible solution: Graphical representation



Situation: Set of constraints does not form a feasible region at all due to inconsistency in the constraints

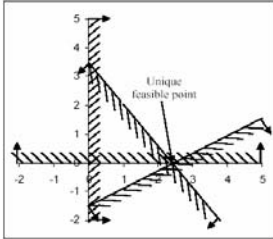
Solution: Optimal solution is not feasible

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Unique feasible point: Graphical representation



Situation: Feasible region consist of a single point. Number of constraints should be at least equal to the number of decision variables

Solution: There is no need for optimization as there is only one feasible point

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